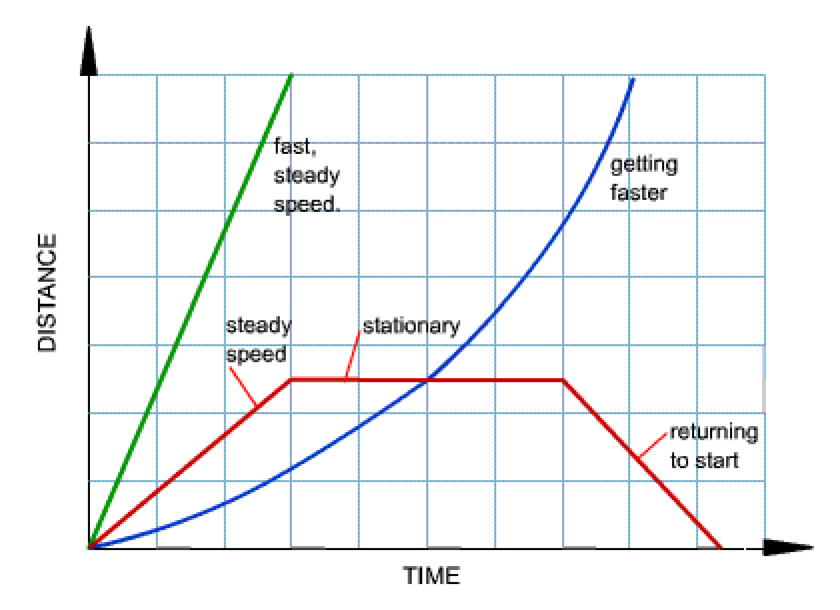
Motion Graphs

It is said that a picture is worth a thousand words. The same can be said for a graph.

Once you learn to read the graphs of the motion of objects, you can tell **at a glance** if the object in question was moving toward you, away from you, speeding up slowing down, or moving at constant speed.



Case Study #1

Imagine you are standing by the side of the road. An automobile approaches you from behind, traveling at a constant speed.

Then the car passes you, disappearing off into the distance.

To graph the motion of this car, you will need some data.

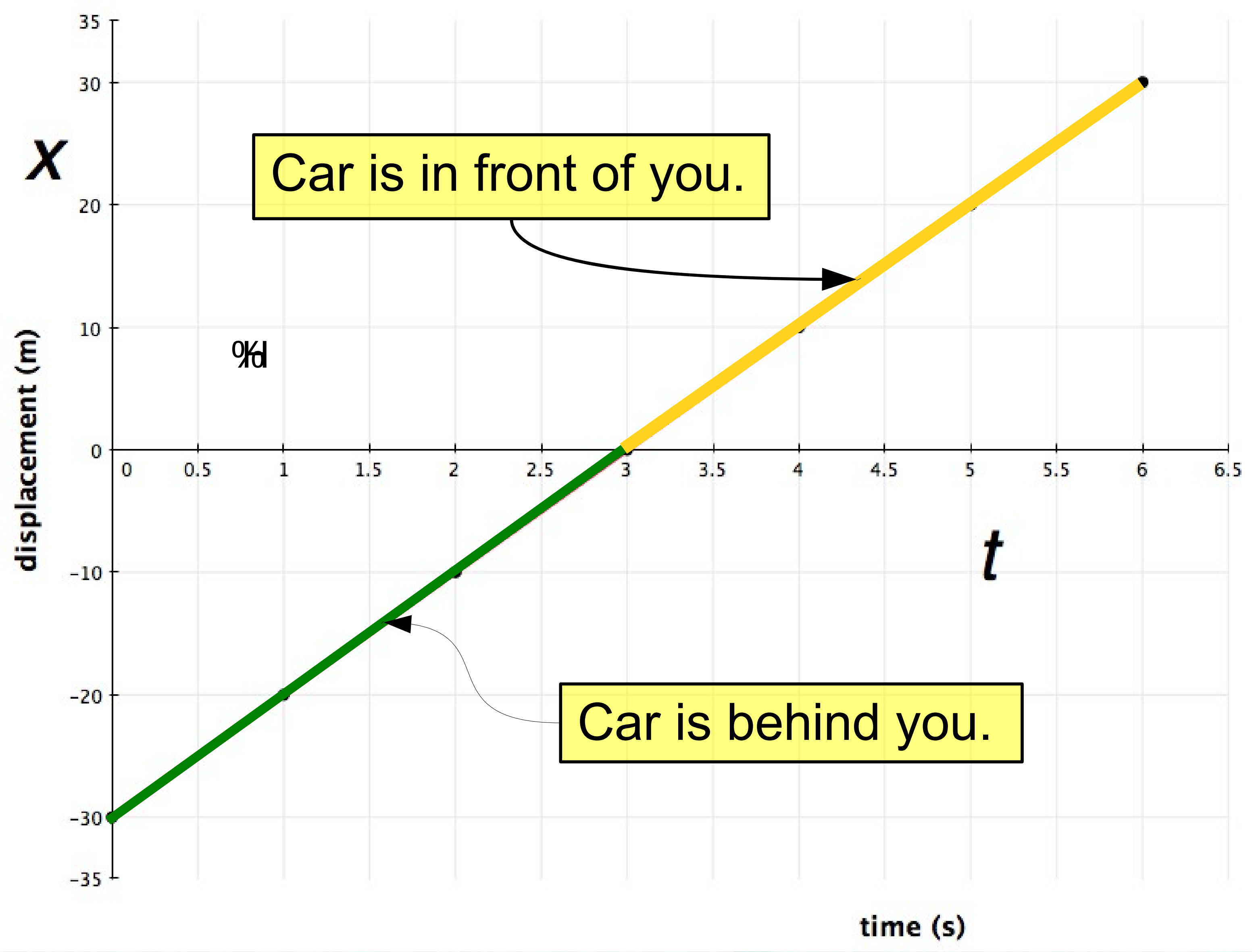
The Data Table

Using a stopwatch and roadside markers, you keep track of how far the car is from you (it's displacement) at each moment in time.

Negative displacements show that the car was behind you. Positive displacements show that the car was in front of you.

time (s)	displacement (m)
0	-30
1	-20
2	-10
3	0
4	10
5	20
6	30

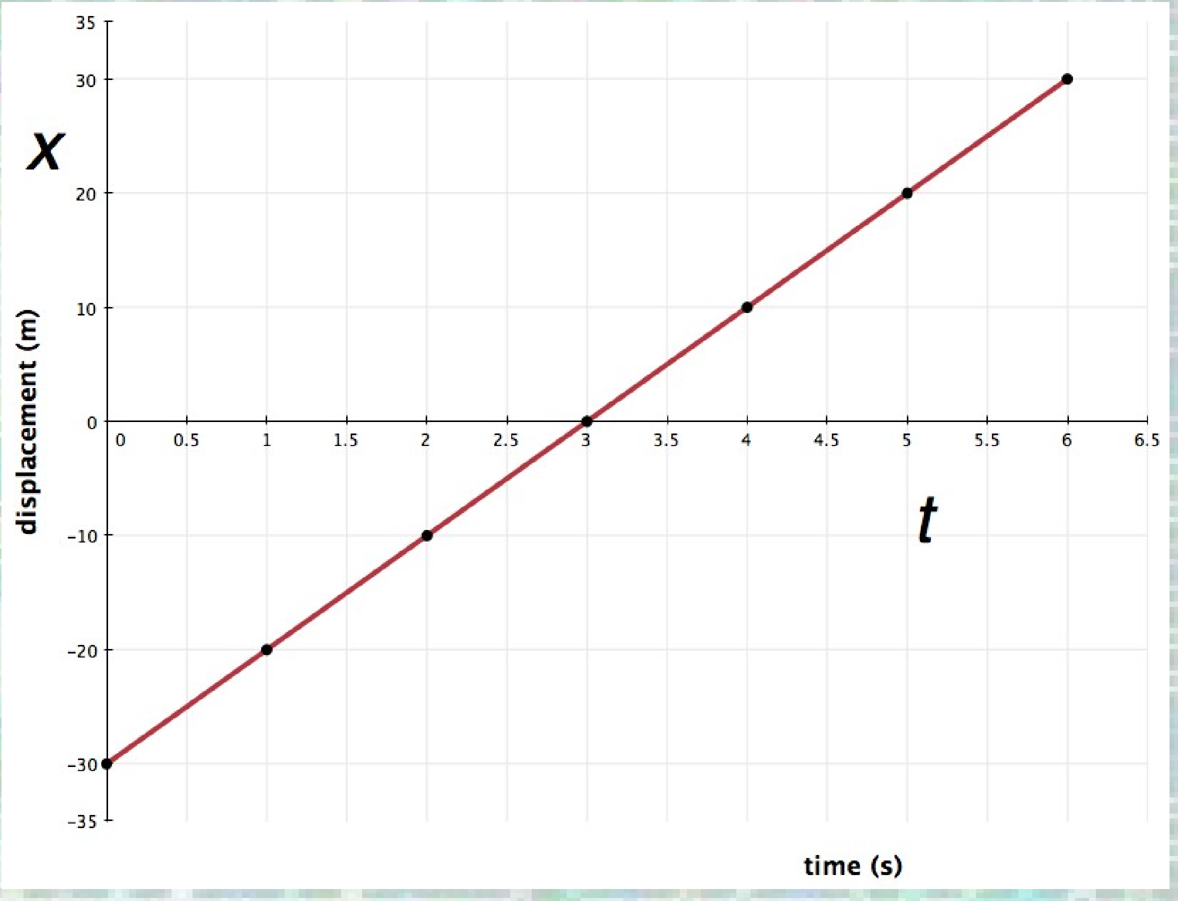
Graph for Case Study #1



Slope of Graph

The slope of a displacement vs. time graph tells us the object's velocity.

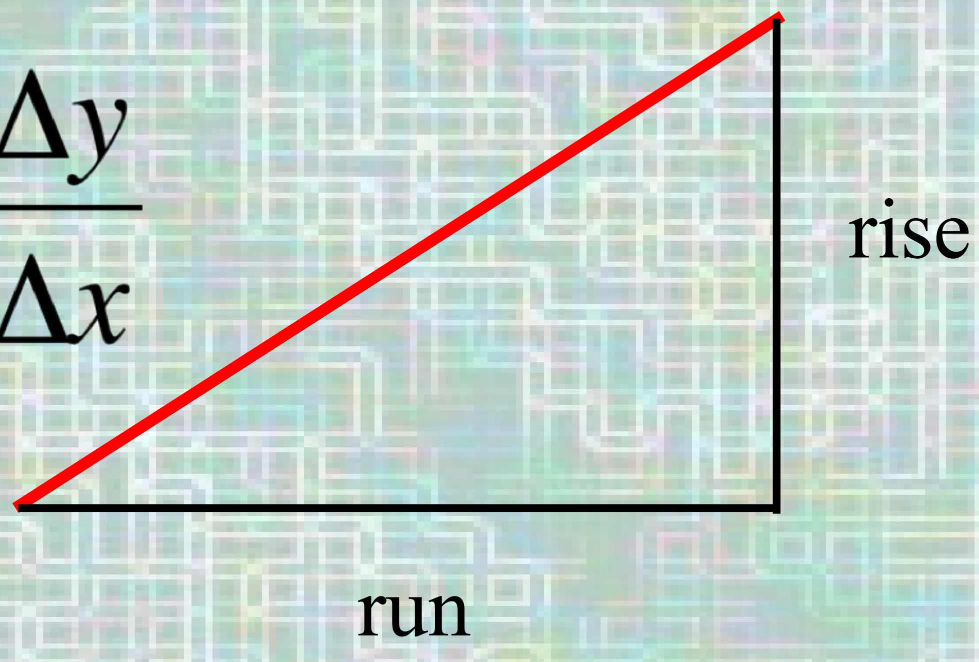
In this case, the line is straight, meaning that the slope (and hence, the velocity) remains constant for this automobile.



Calculating the Slope

In mathematics, the slope of a graph is expressed as the rise over the run.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$



Slope as Velocity

In this particular example, we have put time (Δt) on the horizontal axis and displacement (Δx) on the vertical axis.

The equation now becomes:

$$\text{slope} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o}$$

Finding the Velocity of Our Car

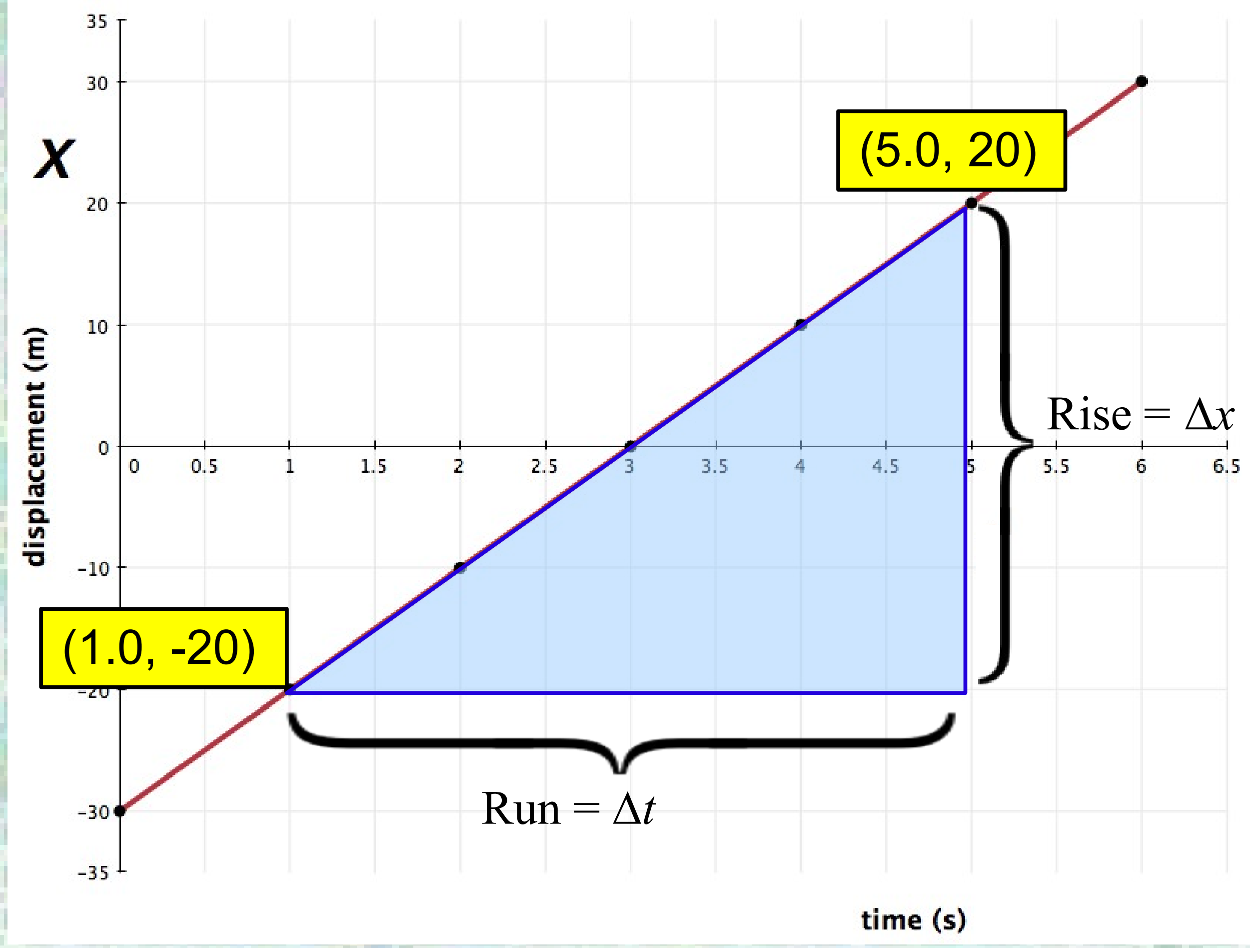
Laying out a triangle on our graph helps us find the values for the **rise** and the **run**.

$$\text{slope} = \frac{x_f - x_o}{t_f - t_o}$$

$$\text{slope} = \frac{20\ \text{m} - (-20\ \text{m})}{5.0\ \text{s} - 1.0\ \text{s}}$$

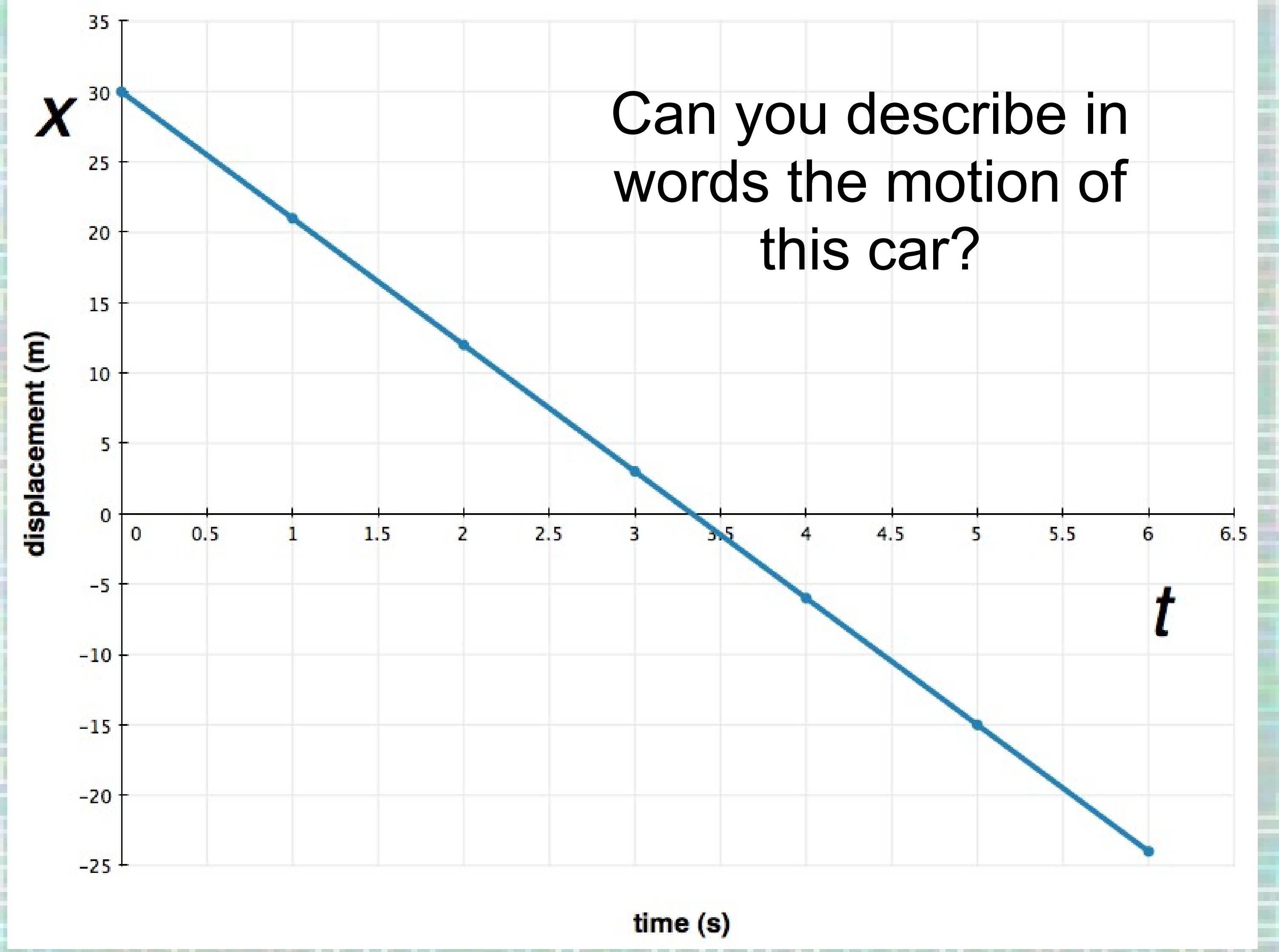
$$\text{slope} = \frac{40\ \text{m}}{4.0\ \text{s}}$$

$$\text{slope} = \bar{v} = 10\ \text{m/s}$$



Case Study #2

Here's a graph that describes the motion of another car you see while standing along the side of the road.

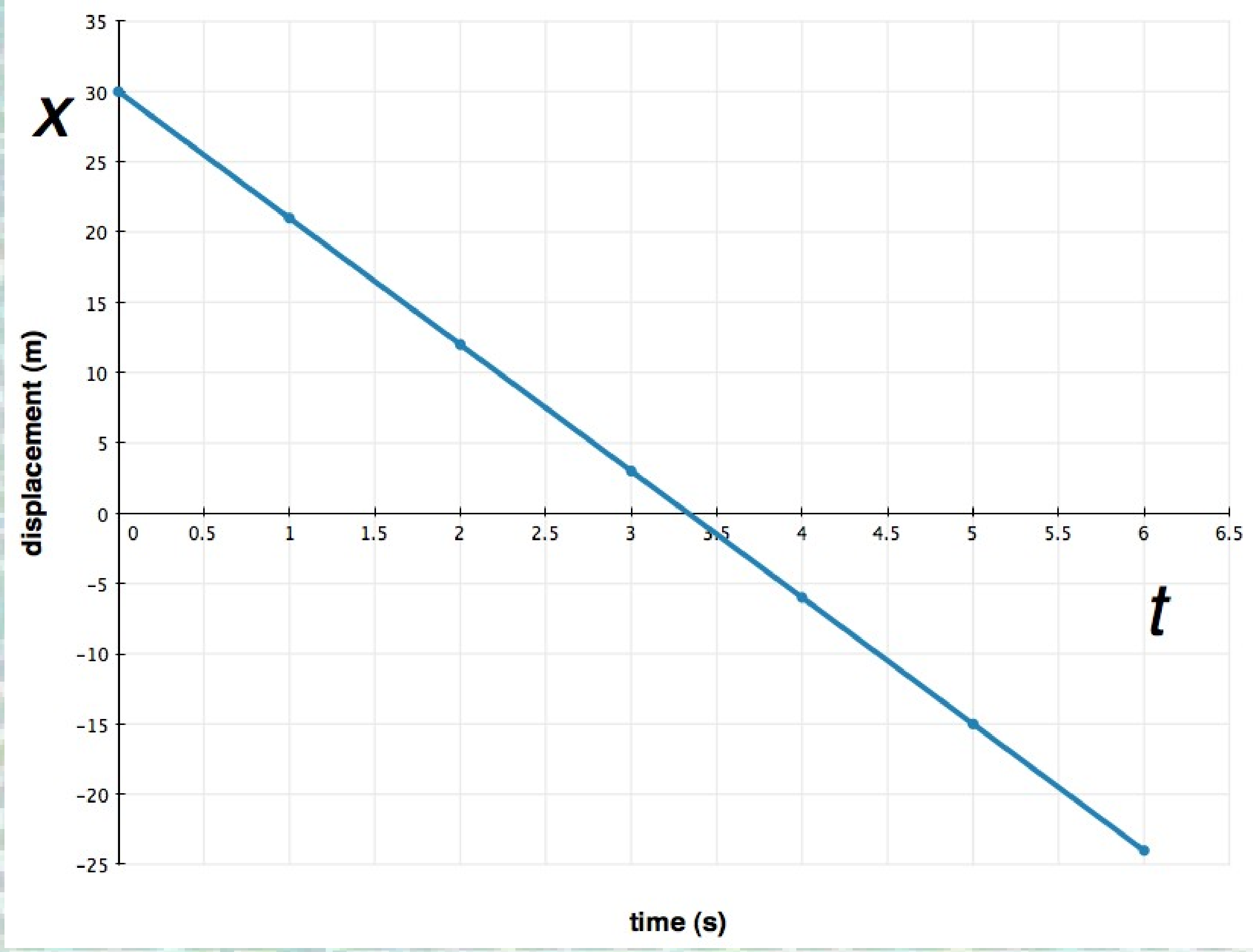


Case Study #2

If you said that this car approached you from **in front**, and then passed on by you, you were correct!

It is also true that this car is traveling at a constant velocity (the graph line is straight.)

But *is it the same* constant velocity the last car had?



Velocity from Slope

It is the same velocity *if* the slope of the graph line is the same.

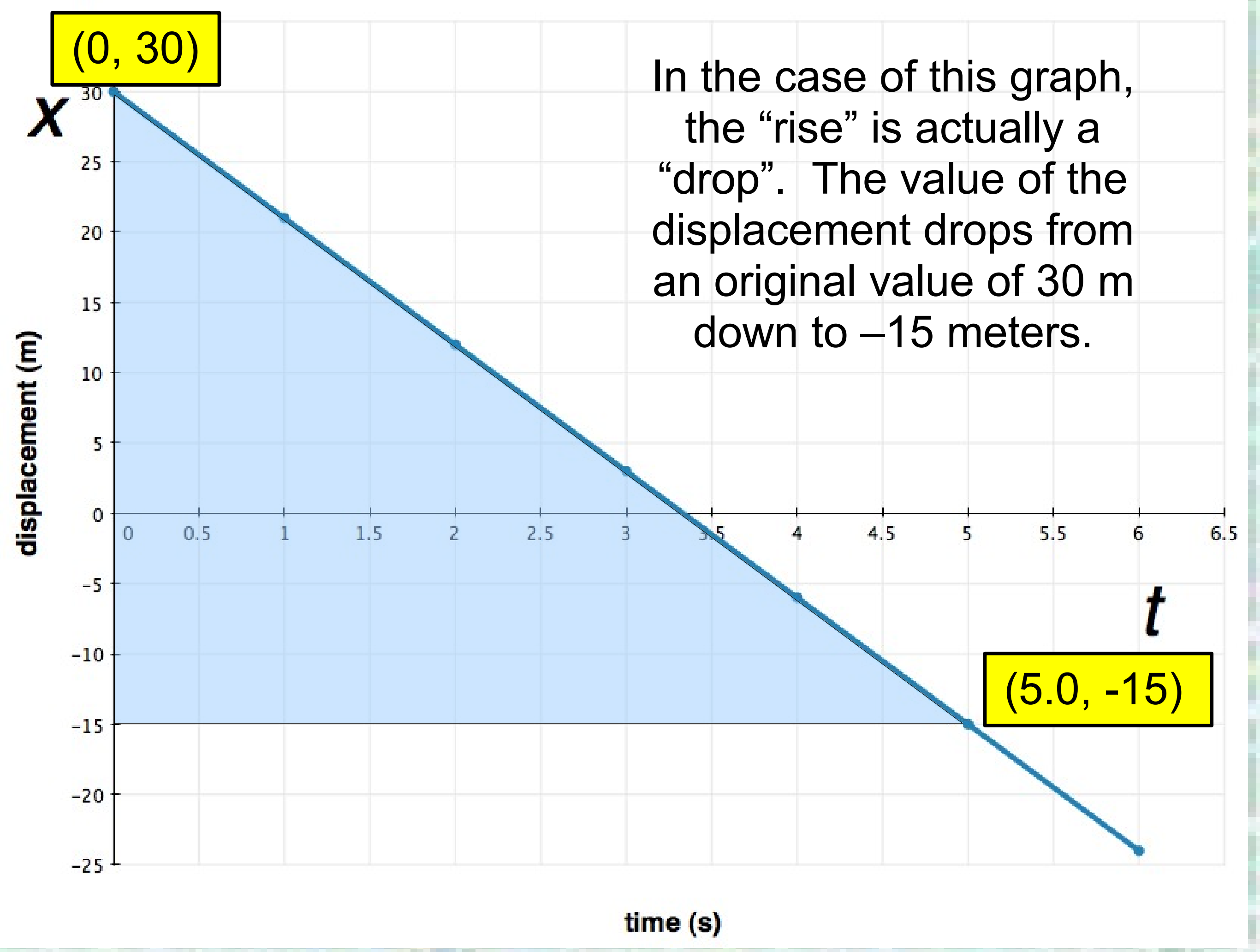
To find out, we must calculate the slope.

$$\text{slope} = \frac{x_f - x_o}{t_f - t_o}$$

$$\text{slope} = \frac{(\text{-15 m}) - (+30\text{ m})}{5.0\text{ s} - 0.0\text{ s}}$$

$$\text{slope} = \frac{-45\text{ m}}{5.0\text{ s}}$$

$$\text{slope} = \bar{v} = -9.0\text{ m/s}$$

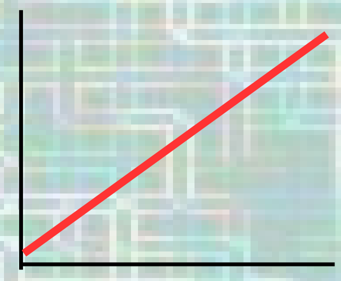


Results for Case Study #2

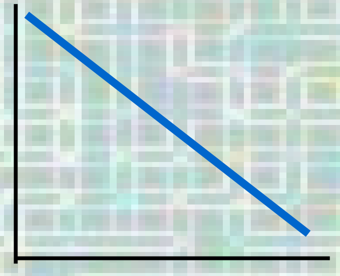
And so we see that this second car is traveling a bit slower than the first one; its velocity coming in with a magnitude of 9.0 m/s instead of 10 m/s.

The negative sign on the second car's velocity means that it was traveling in the opposite direction of the first car.

That's easy to tell at a glance, just by looking at the two graphs.



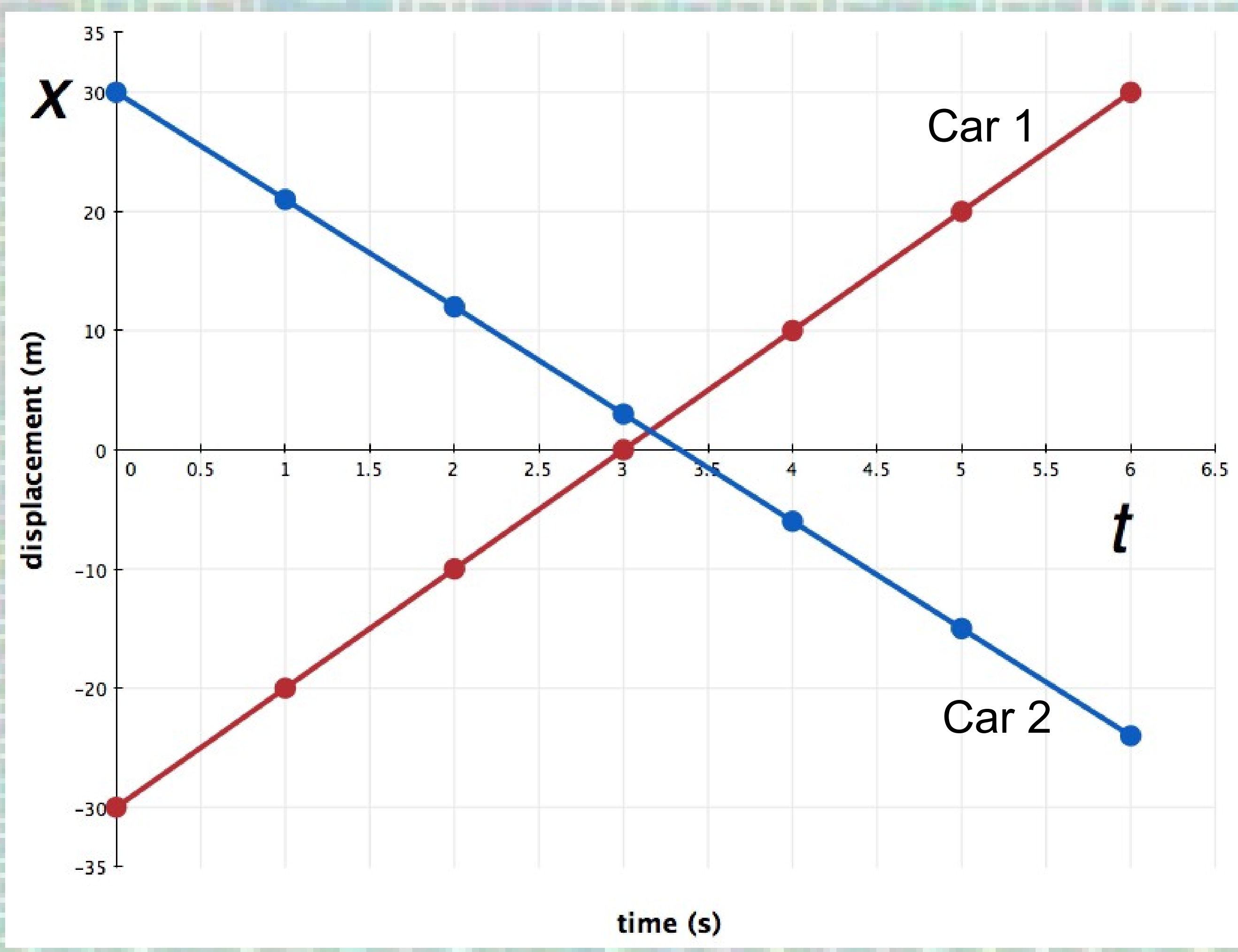
Positive slope (car #1)



Negative slope (car #2)

Graphing Both Cars

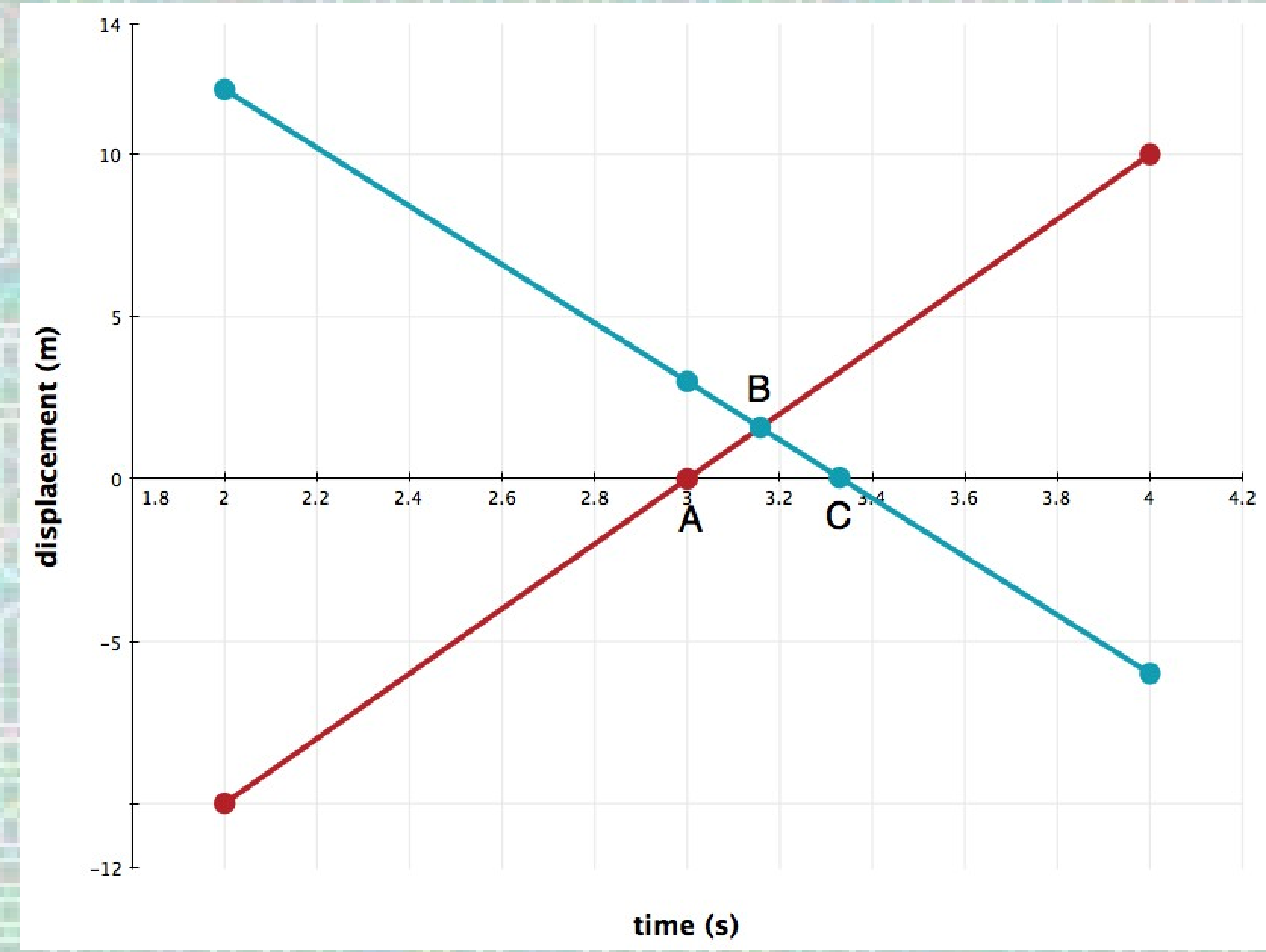
It is much easier to compare the motions of the two cars when you graph those motions on the same set of axes.



Intersections and Intercepts

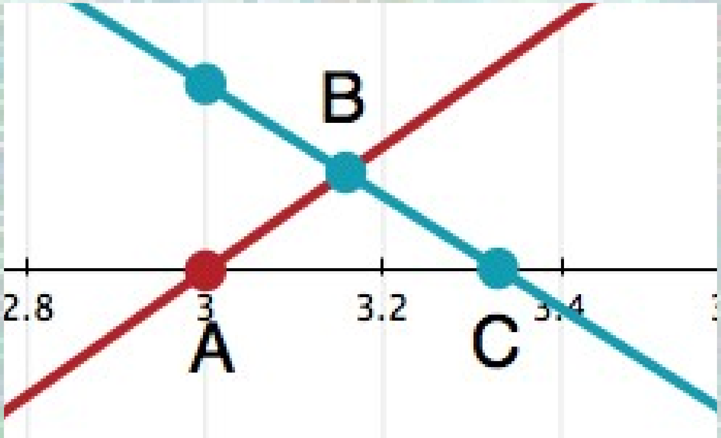
Of particular interest is the center section of the graph.

Can you figure out what's happening at each of the labelled points A, B, and C?



Intersections and Intercepts

Point A is where the first car (that came from behind) passes you.



Point B is where the two cars pass each other (in front of you).

Point C is where the second car (that came from in front of you) passes you by.

Where graph lines intersect the horizontal axis is where the objects pass by the observer.

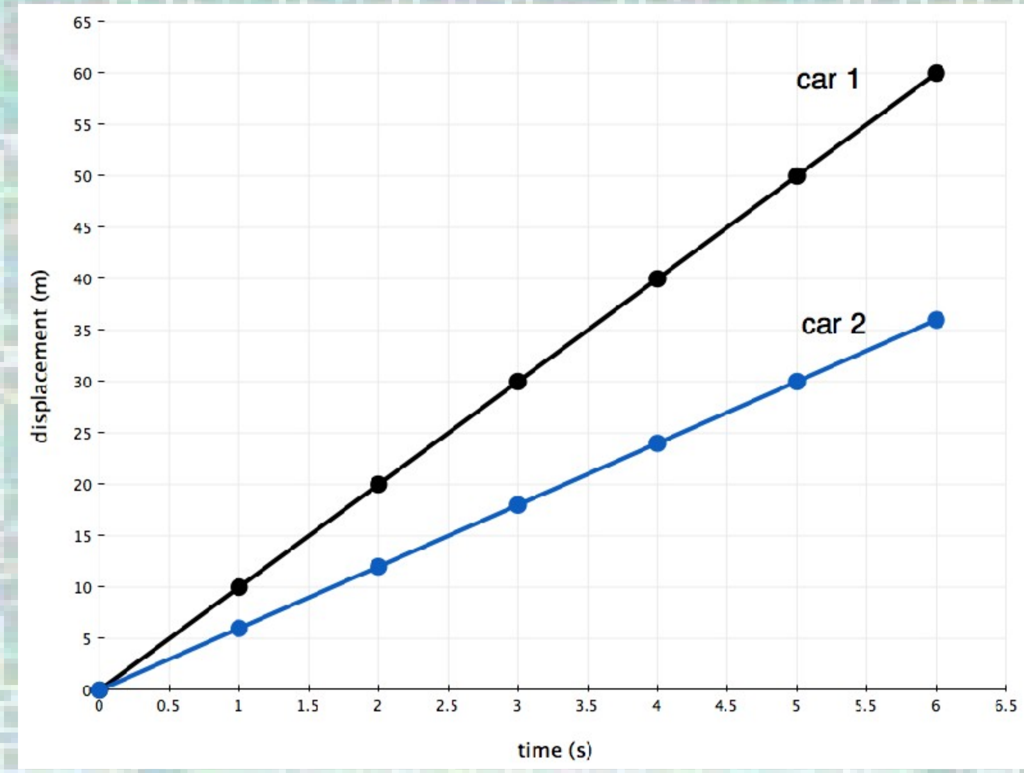
Where graph lines intersect each other is where the objects pass each other.

Moving in the Same Direction

When two cars are moving in the same direction, it is much easier to compare their speeds, both in real life, and on a graph.

The slopes of their graph lines are obviously different. The slope for car 1 is steeper, therefore it must be going faster.

For practice, try calculating the slope of each car's graph line.



Check Your Work!

Here are the velocities, calculated for each of the two cars, from the slopes of their graph lines.

For Car 1

$$\text{slope} = \bar{v} = \frac{x_f - x_o}{t_f - t_o}$$

$$\bar{v} = \frac{50\text{ m} - 20\text{ m}}{5.0\text{ s} - 2.0\text{ s}}$$

$$\bar{v} = \frac{30\text{ m}}{3.0\text{ s}} = 10\text{ m/s}$$

For Car 2

$$\text{slope} = \bar{v} = \frac{x_f - x_o}{t_f - t_o}$$

$$\bar{v} = \frac{30\text{ m} - 12\text{ m}}{5.0\text{ s} - 2.0\text{ s}}$$

$$\bar{v} = \frac{18\text{ m}}{3.0\text{ s}} = 6\text{ m/s}$$

Case Study #4

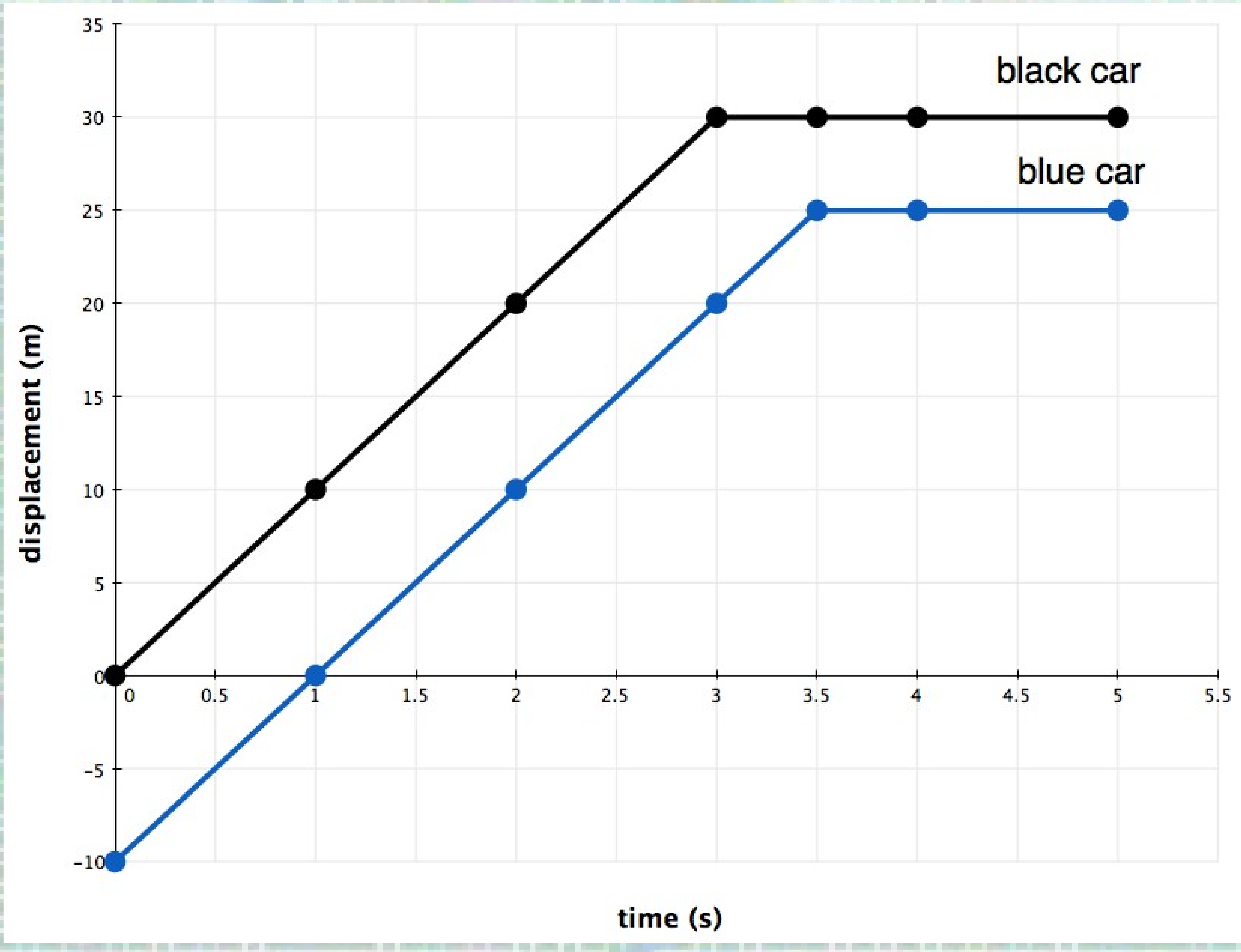
A black car is traveling along the highway at constant velocity. A blue car follows along behind it, at the same velocity.

The black car pulls to the side of the road and stops. The blue car pulls up behind it and also stops.

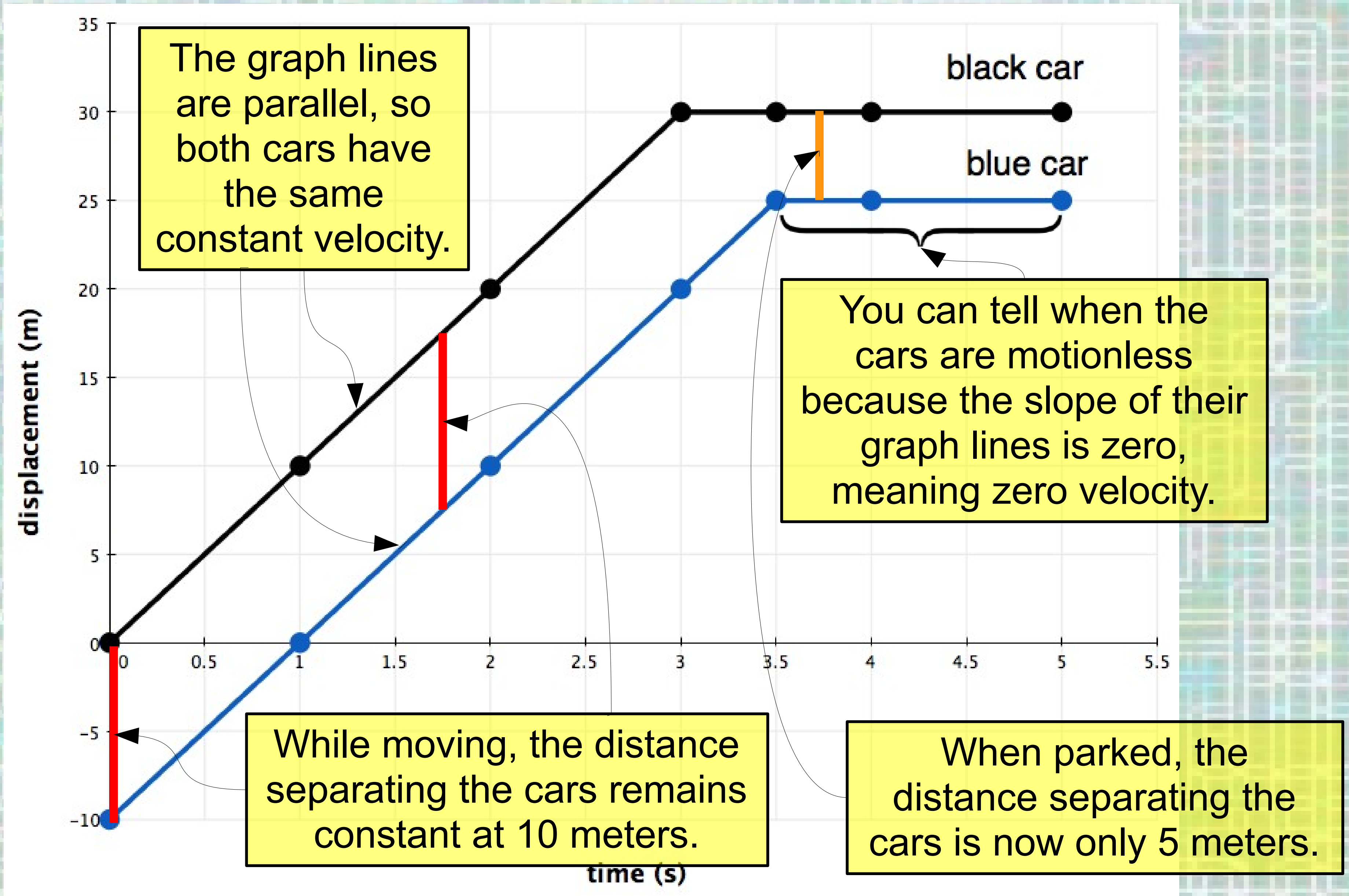
First, we look at the data table that describes their motion, then we'll graph it.

time (s)	Displacement black car (m)	Displacement blue car (m)
0	0	-10
1	10	0
2	20	10
3	30	20
3.5	30	25
4	30	25
5	30	25

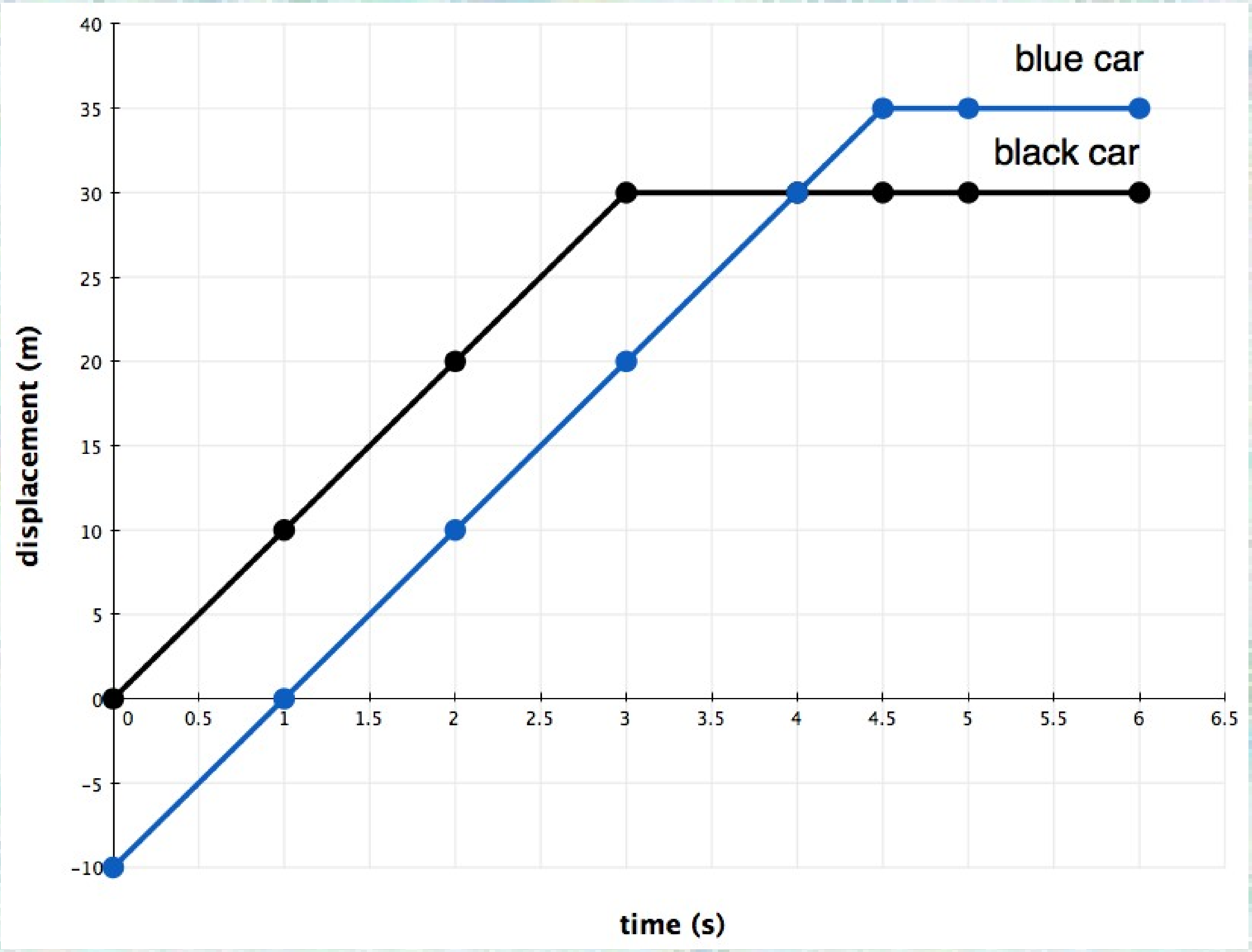
Case Study #4


black car
blue car
displacement (m)
time (s)
35
30
25
20
15
10
5
0
-5
-10
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5

Reading Information off the Graph



What's Different About This One?

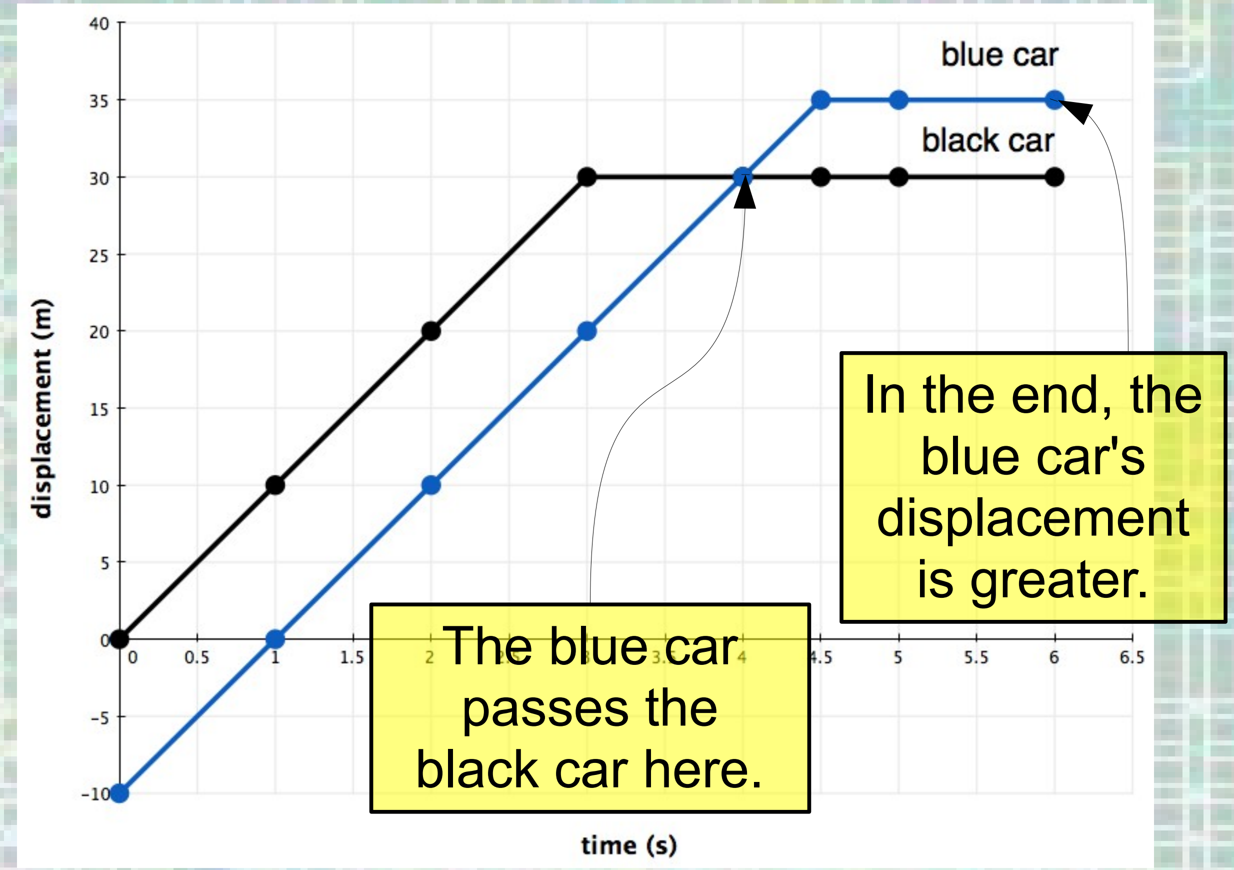


Reading the Graph

As before, the blue car followed the black car while traveling along the highway at constant velocity. Just as before, the black car pulled over and stopped.

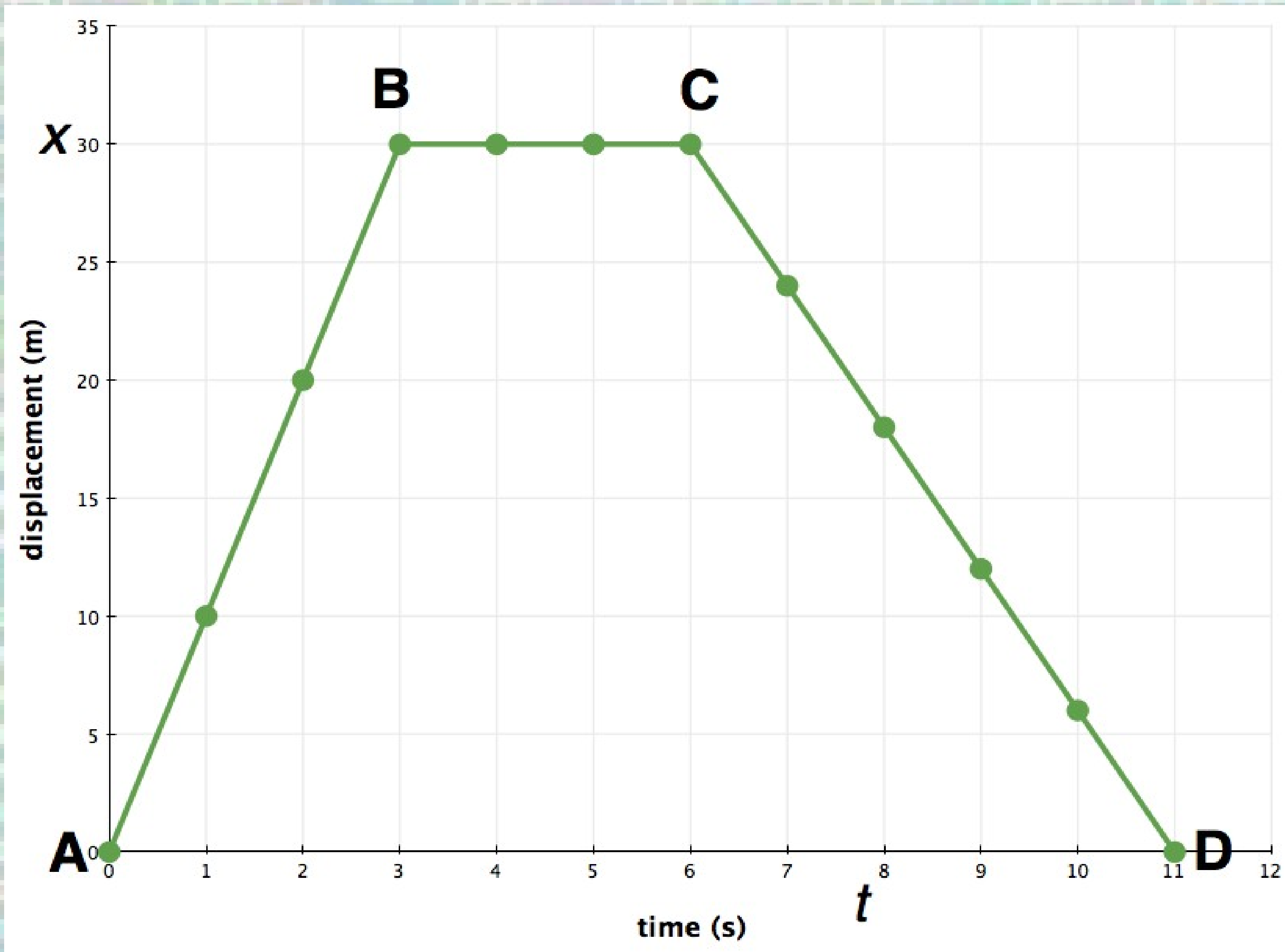
What's different is that this time, the blue car passed the black car and then pulled over to stop *in front of it*.

We know this because, in the end, the blue car is farther out in front.

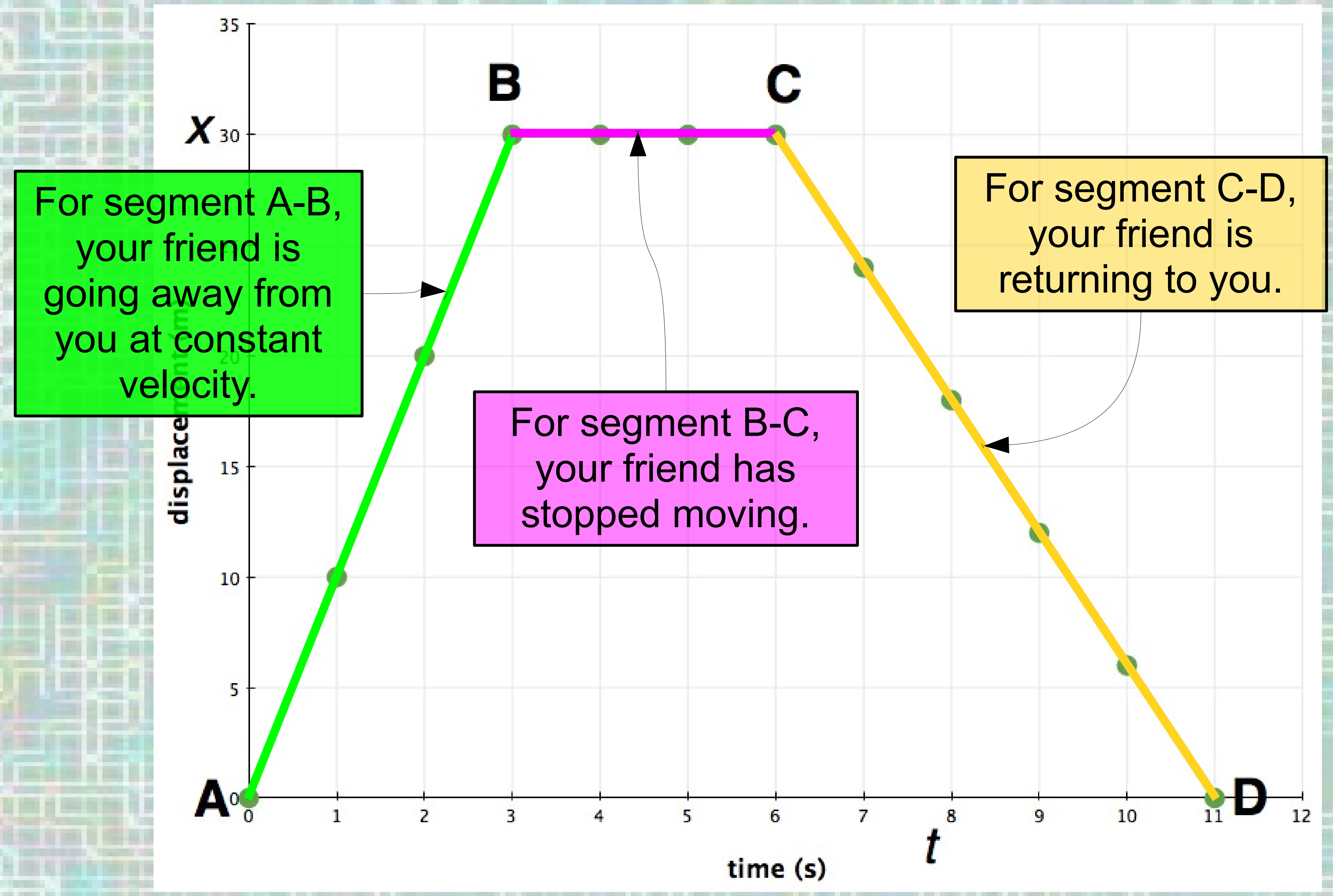


The New Motorcycle

Your friend just bought a new motorcycle and wants to show it off for you. Look at the graph below, and see if you can describe the motion of your friend on his new machine.



Describing the Motion



Quantifying the Results

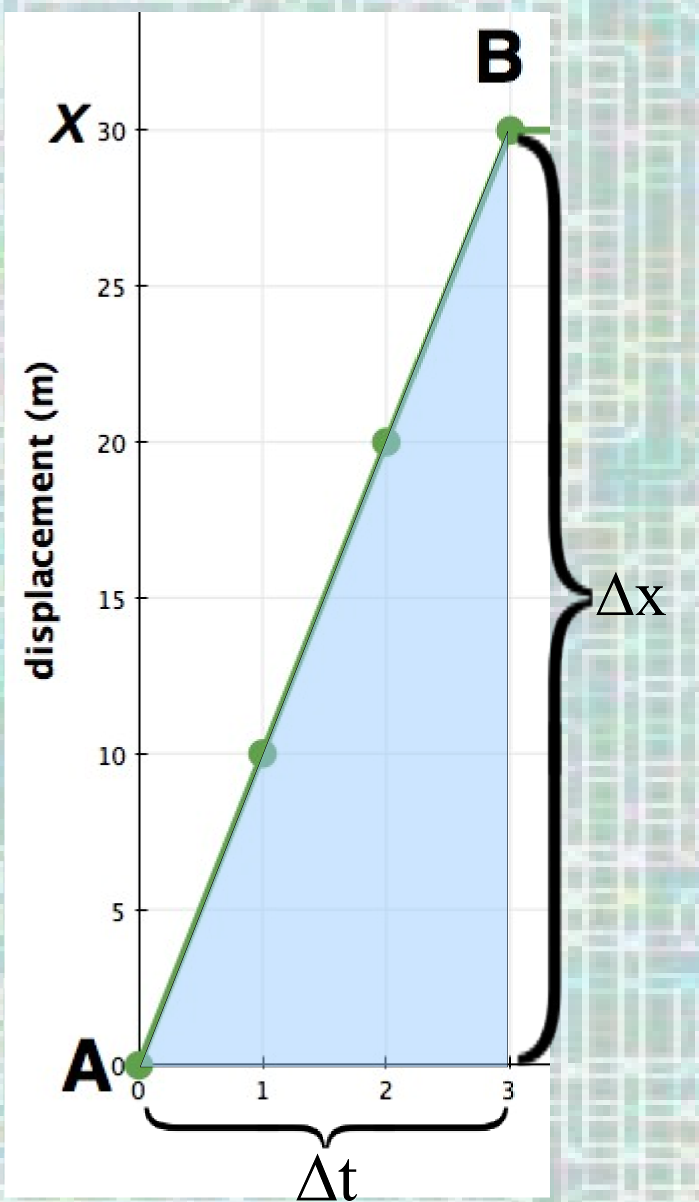
Because we have grid lines and numbers on the axes, we can include numerical values in our descriptions.

During segment A-B, we can see that your friend travels 30 meters out in front of you.

$$\text{slope} = \bar{v} = \frac{x_f - x_o}{t_f - t_o}$$

$$\bar{v} = \frac{30\text{ m} - 0\text{ m}}{3\text{ s} - 0\text{ s}} = 10\text{ m/s}$$

The slope calculation tells us how fast he was going.



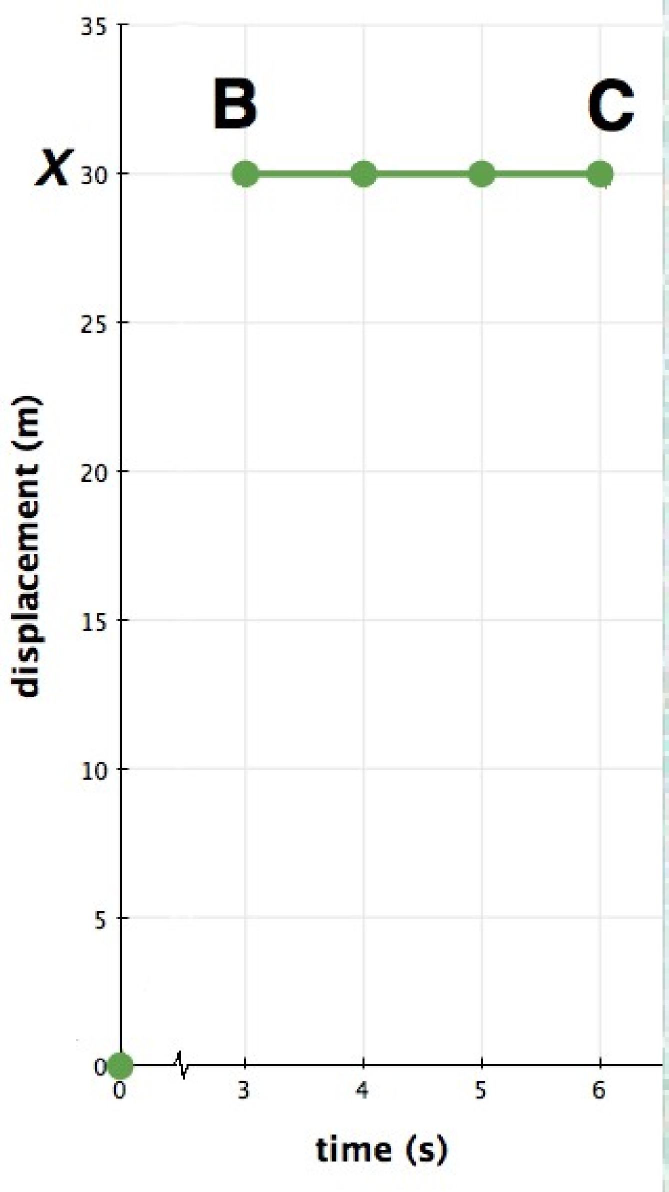
Quantifying the Results

For segment B-C, your friend remains at a distance of 30 meters in front of you for 3 seconds.

He's motionless. The slope of the graph line is quite obviously zero, but if you had to show your work, here's how you'd do it.

$$\text{slope} = \bar{v} = \frac{x_f - x_o}{t_f - t_o}$$

$$\bar{v} = \frac{30\text{ m} - 30\text{ m}}{6\text{ s} - 3\text{ s}} = 0\text{ m/s}$$



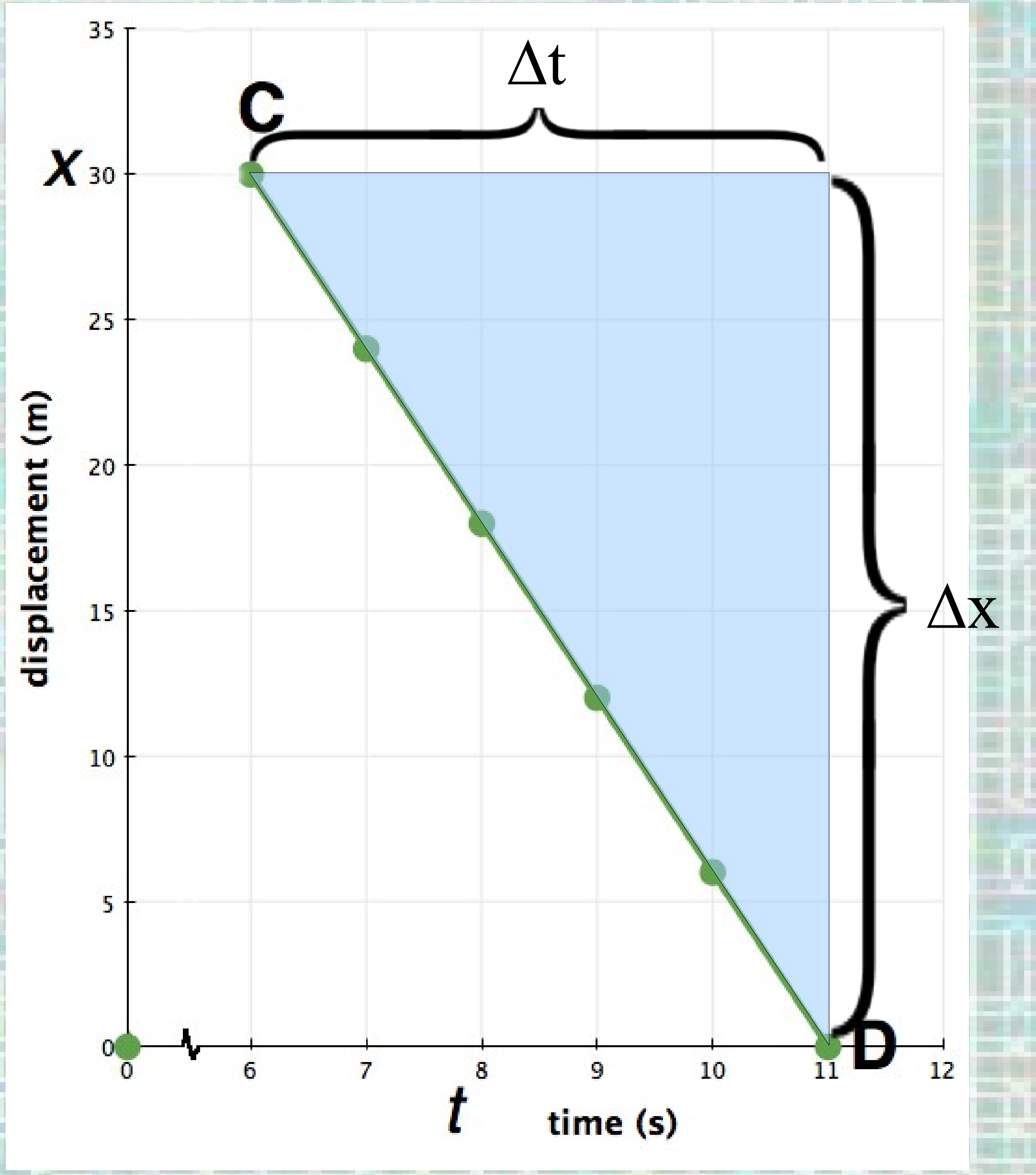
Quantifying the Results

For the C-D segment, the distance between you and your friend continuously decreases as he returns to you.

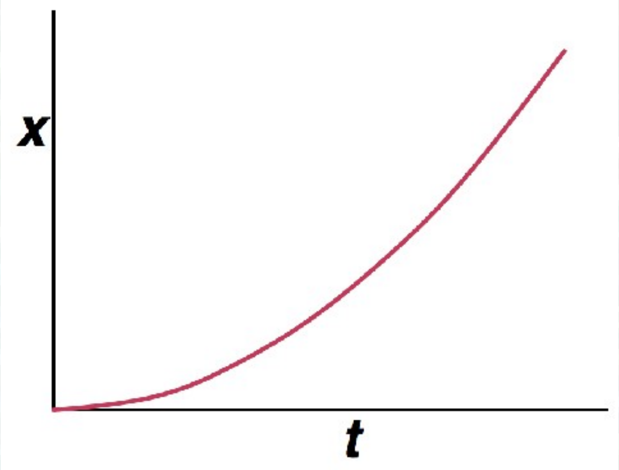
Again, a slope calculation tells us how fast he's going.

$$\text{slope} = \bar{v} = \frac{x_f - x_o}{t_f - t_o}$$

$$\bar{v} = \frac{0\text{ m} - 30\text{ m}}{11\text{ s} - 6\text{ s}} = -6\text{ m/s}$$



Curved Lines Mean Changing Slopes

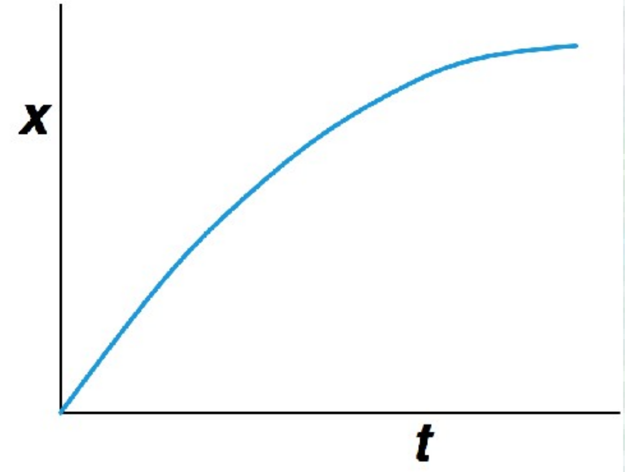


The slope gets steeper with time. This means the object's velocity is increasing.

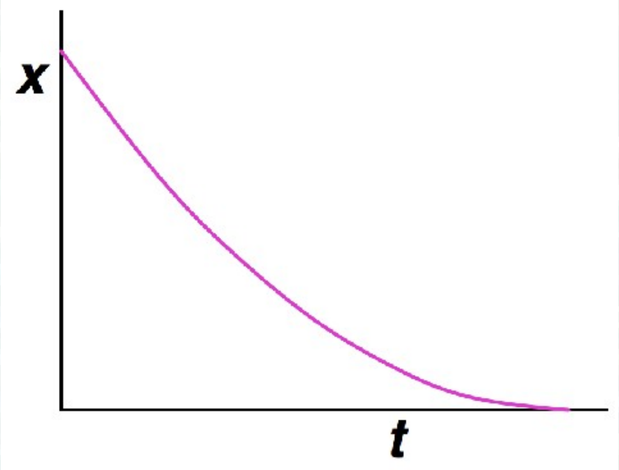
(As before, the increasing displacement means it's going away from you.)

The slope flattens out with time. This means the object's velocity is decreasing, approaching zero.

(As before, the increasing displacement means it's going away from you.)

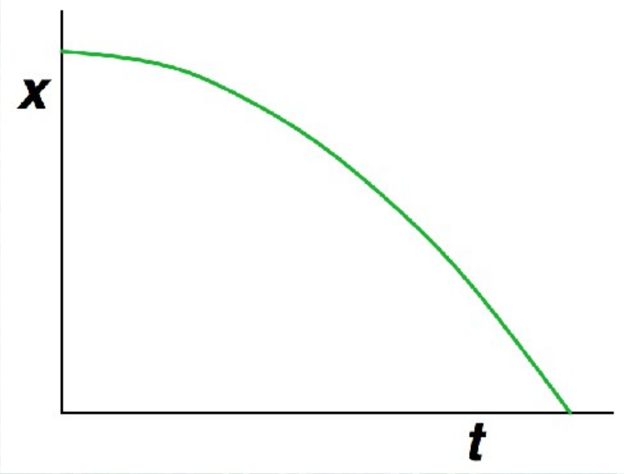


Curved Lines Mean Changing Slopes



The slope flattens out with time. This means the object's velocity is decreasing, approaching zero. (The decreasing displacement means the object is approaching you.)

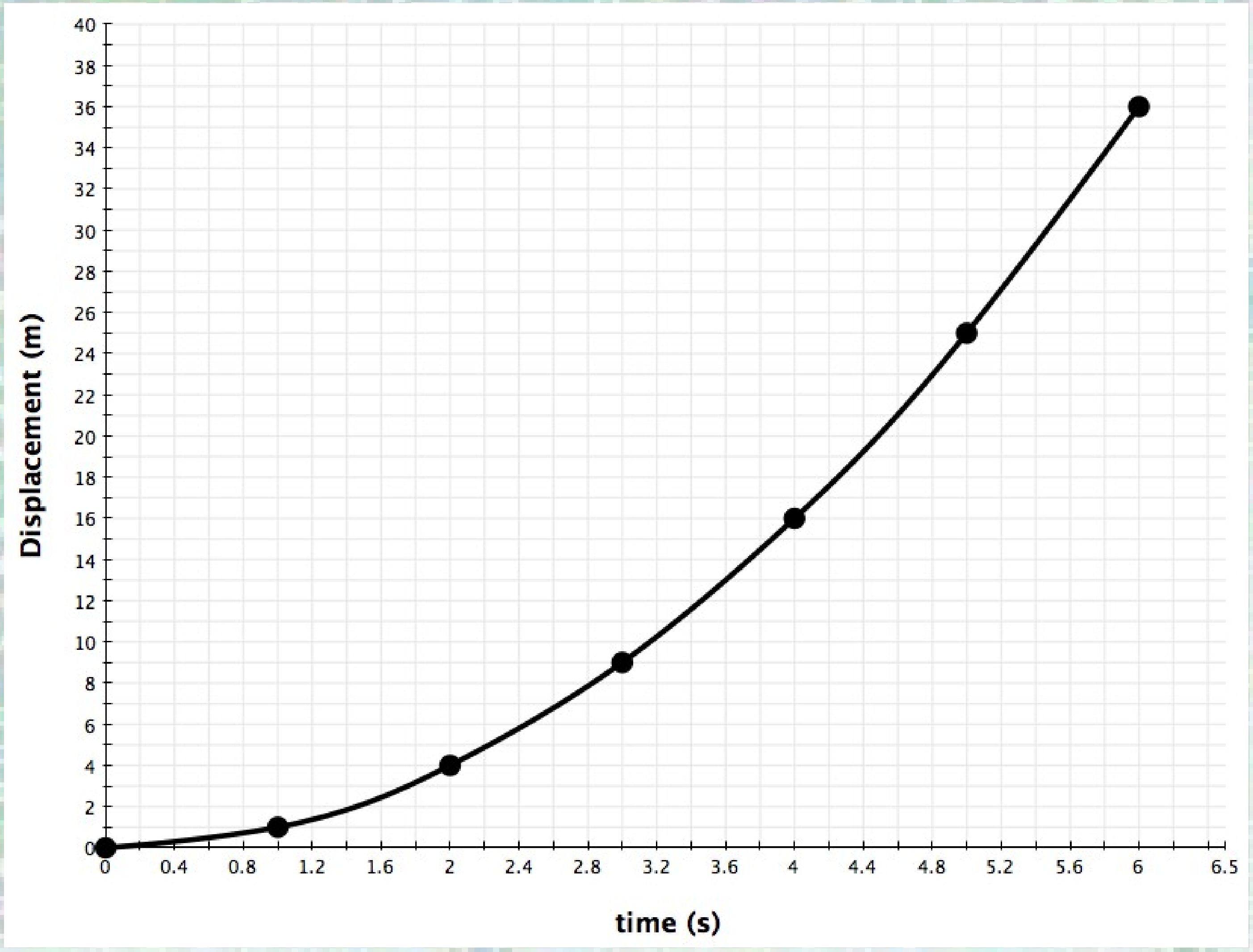
The slope gets steeper with time. This means the object's velocity is increasing. (The decreasing displacement means the object is getting closer to you.)



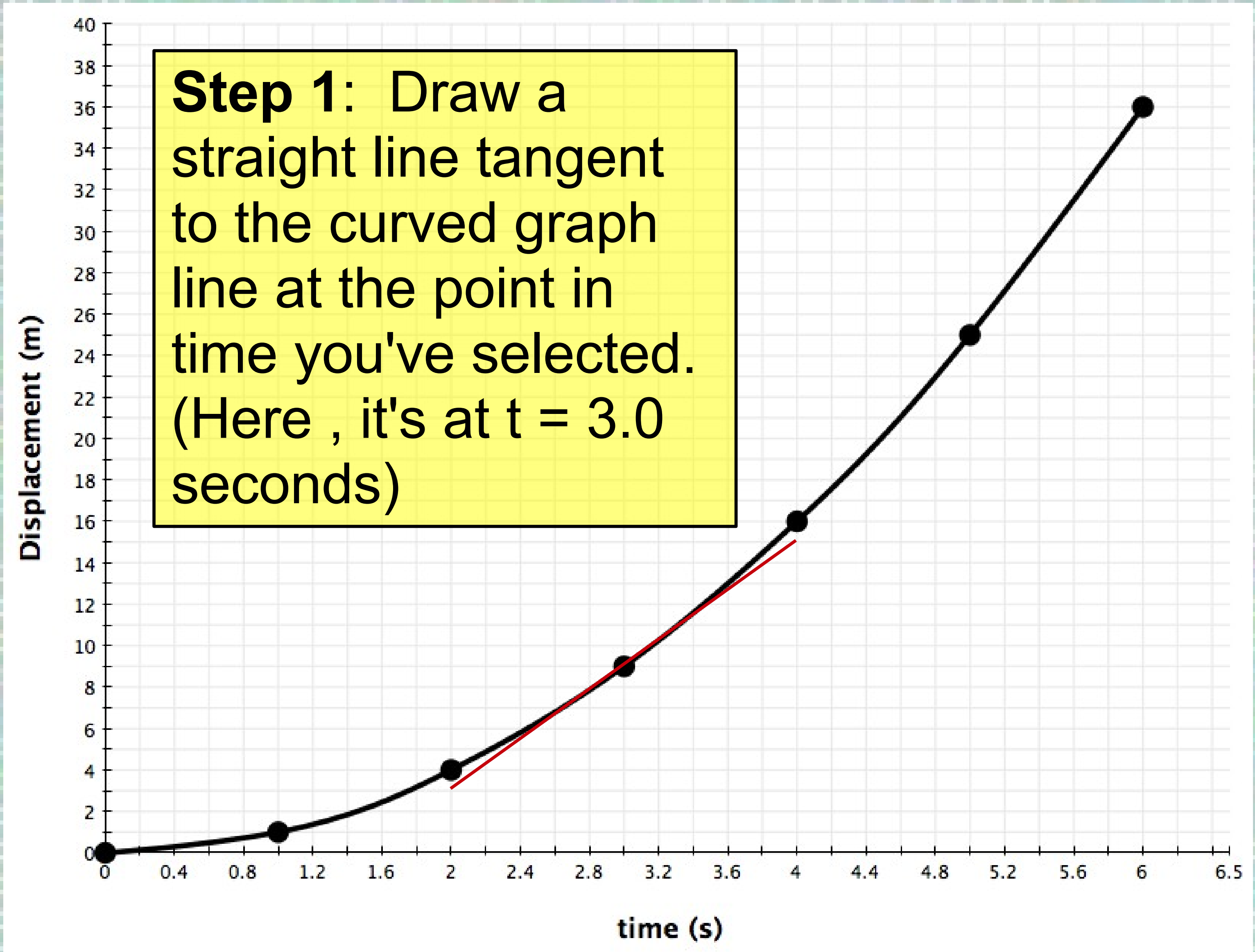
Case Study #5

An object slides down a ramp, picking up speed as it goes.

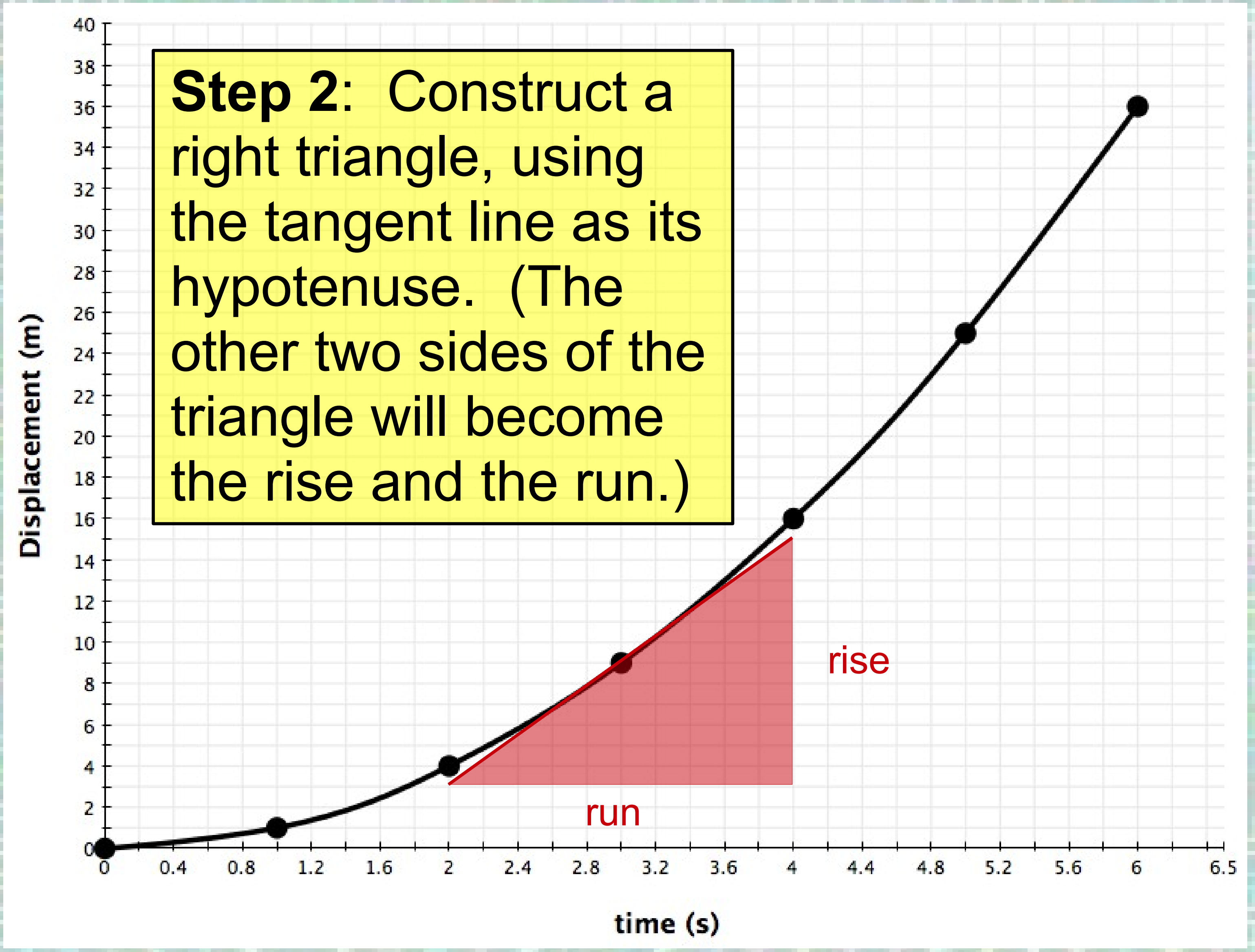
We can figure out how fast the object was going by calculating the slope, but, depending upon which instant in time we pick, we will get a different answer.



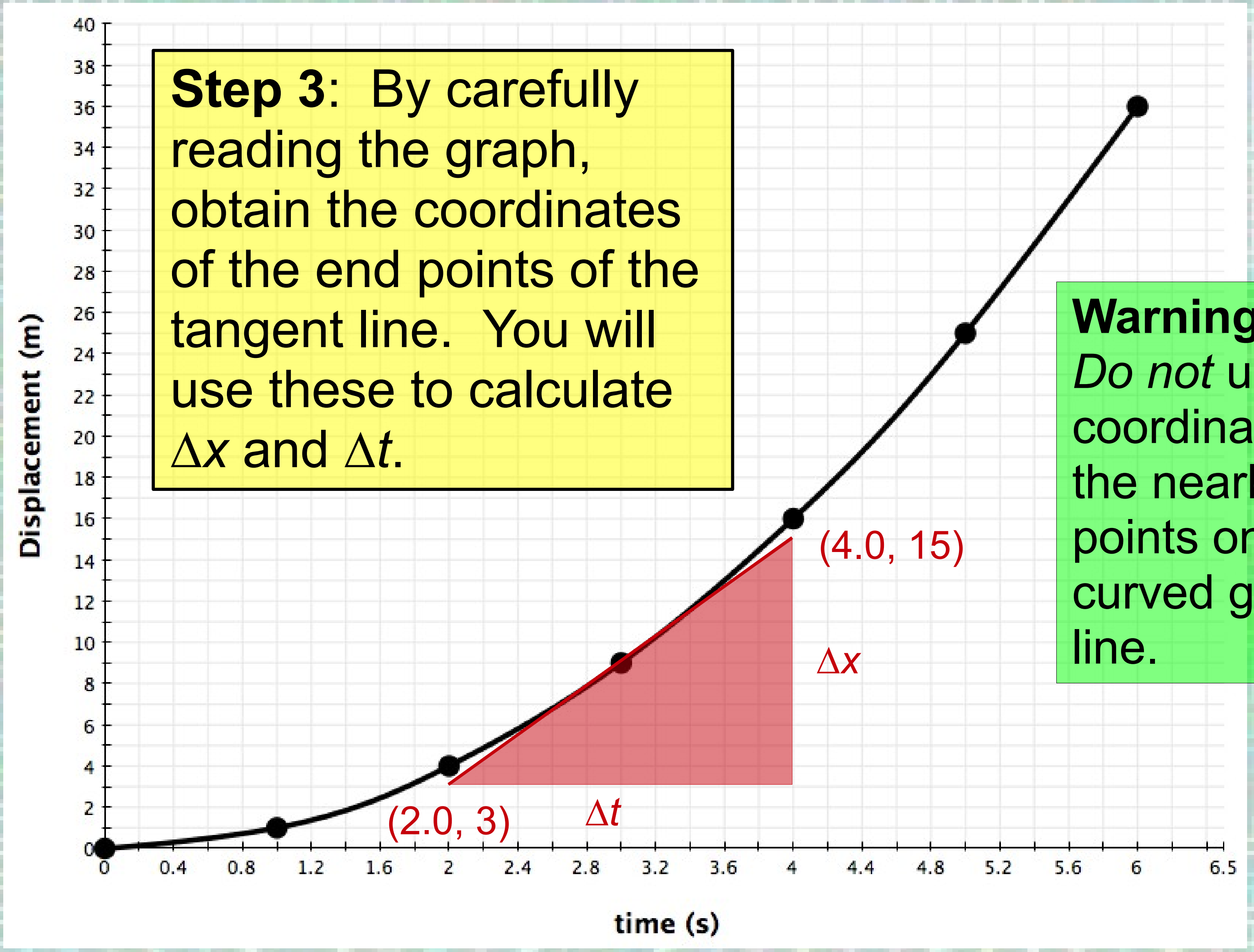
Finding the Velocity (Slope) at a Particular Point



Finding the Velocity (Slope) at a Particular Point



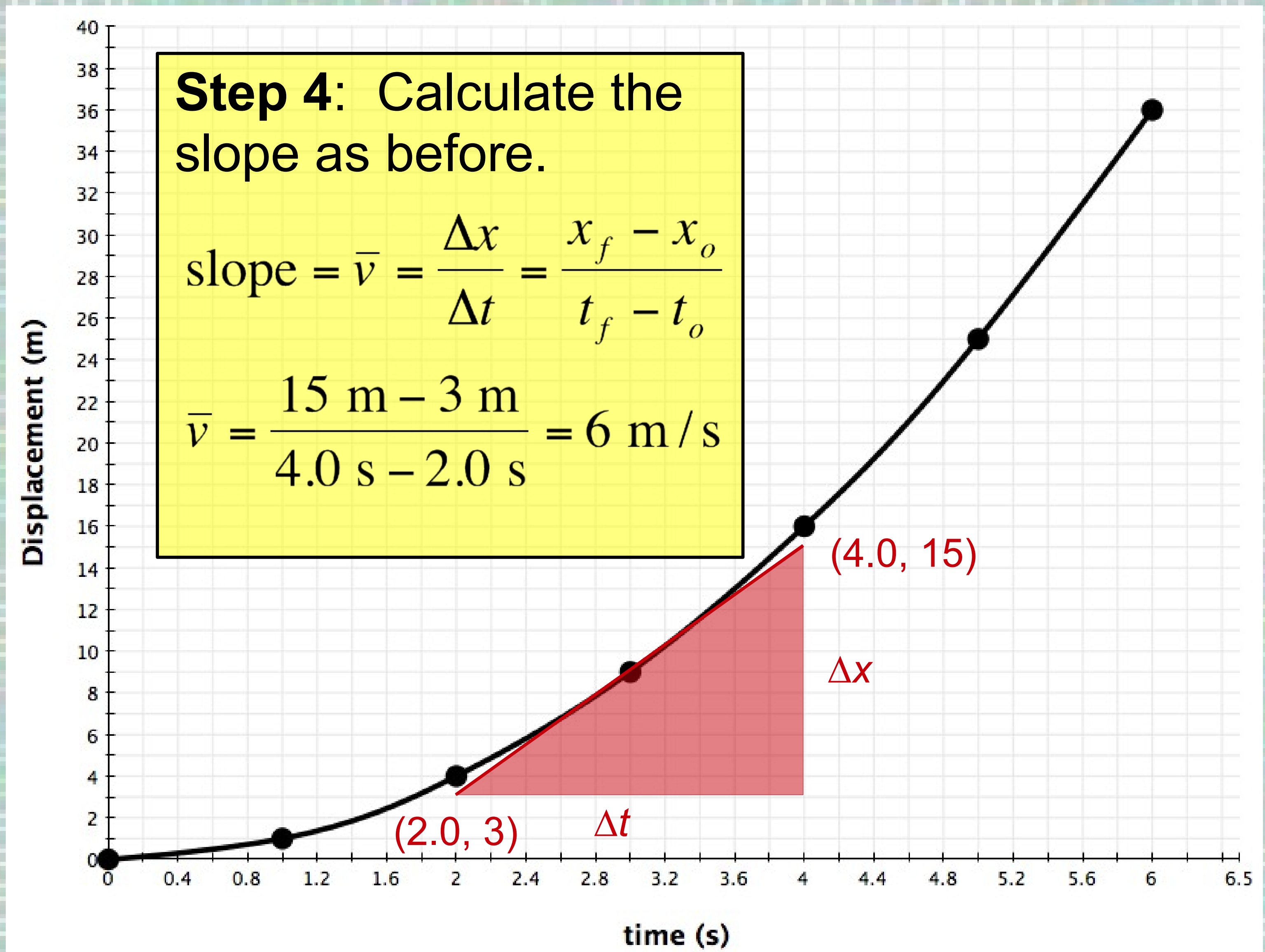
Finding the Velocity (Slope) at a Particular Point



Step 3: By carefully reading the graph, obtain the coordinates of the end points of the tangent line. You will use these to calculate Δx and Δt.

Warning!
Do not use the coordinates of the nearby points on the curved graph line.

Finding the Velocity (Slope) at a Particular Point

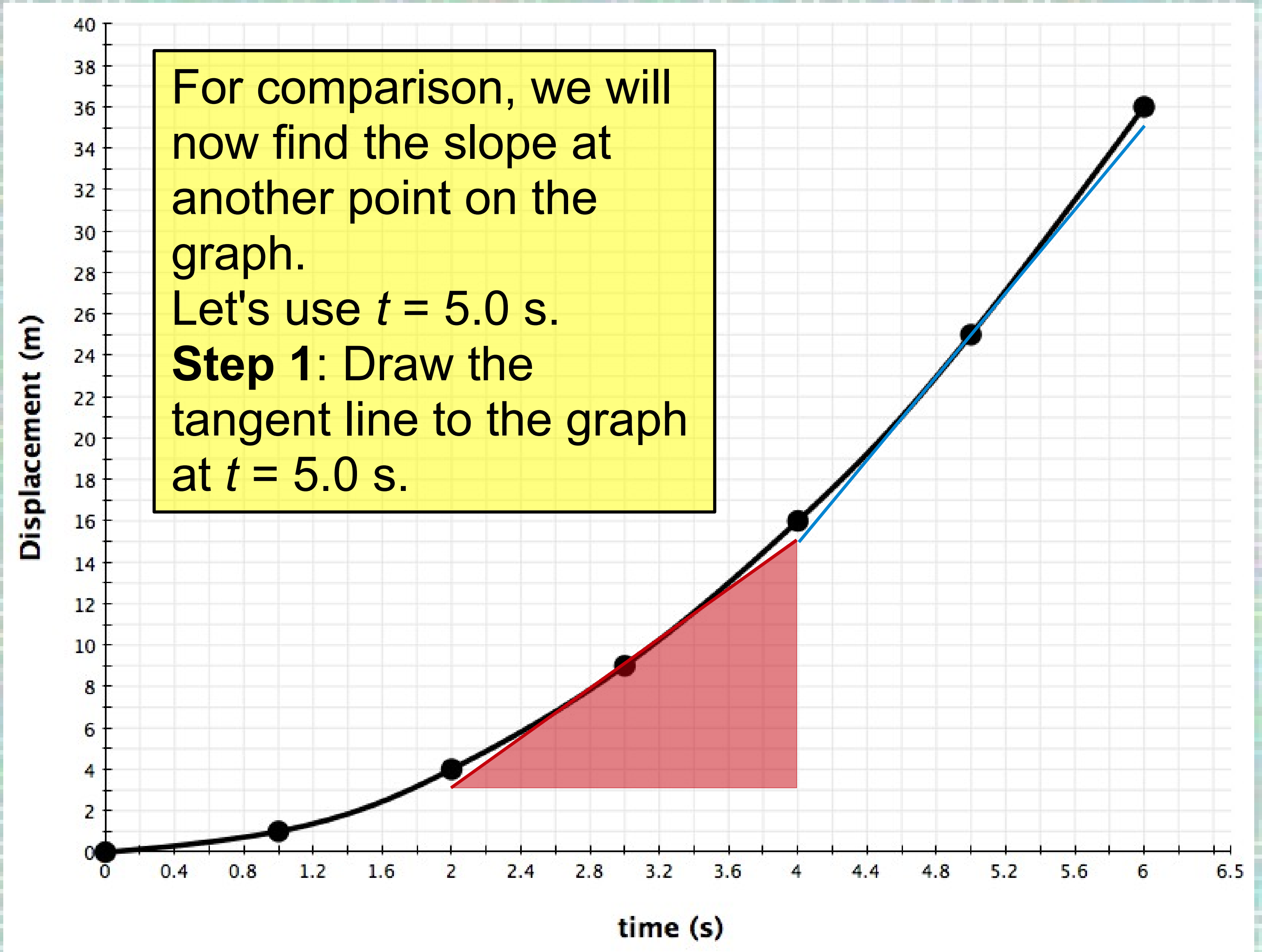


Step 4: Calculate the slope as before.

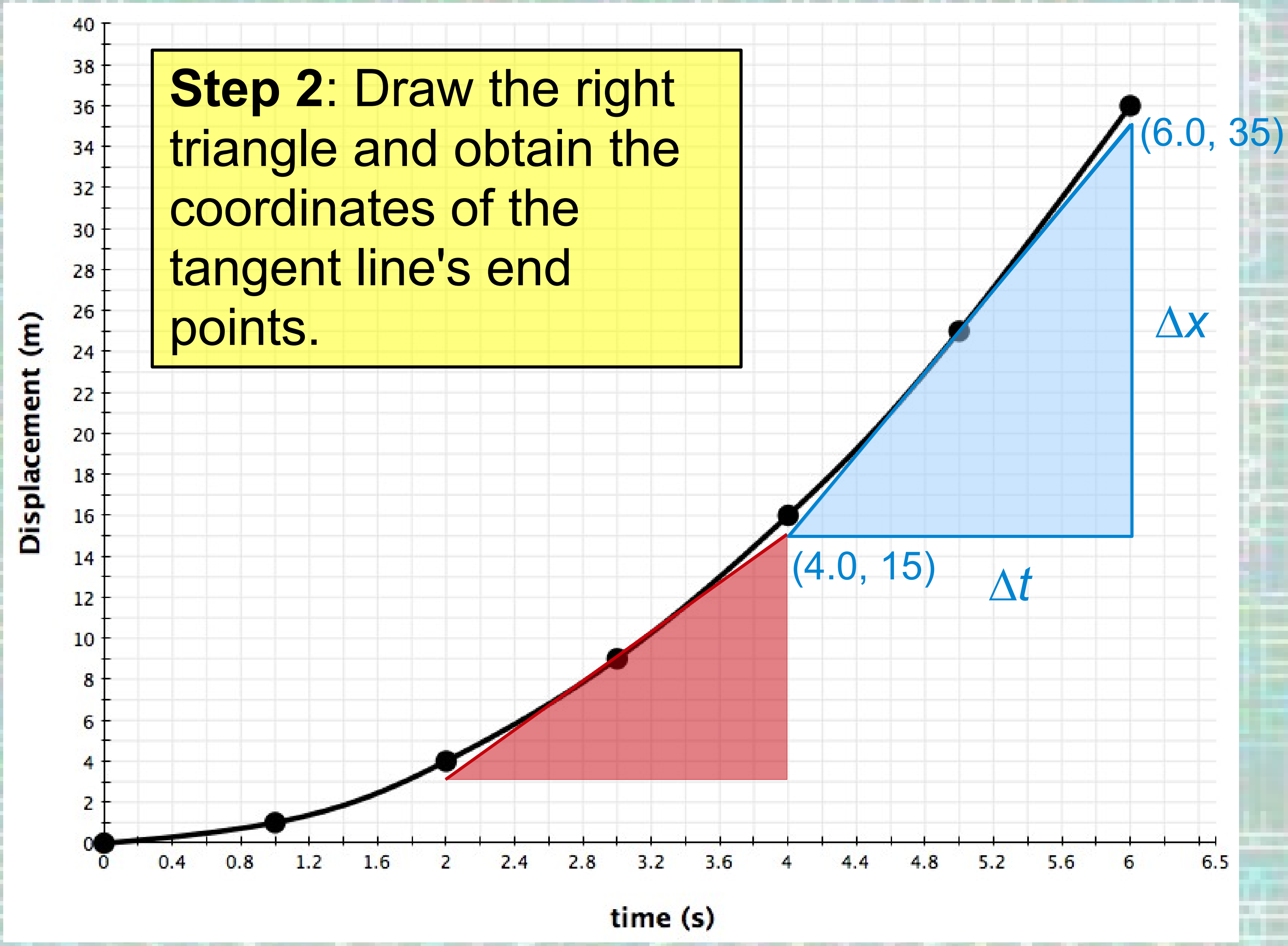
$$\text{slope} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o}$$

$$\bar{v} = \frac{15\text{ m} - 3\text{ m}}{4.0\text{ s} - 2.0\text{ s}} = 6\text{ m/s}$$

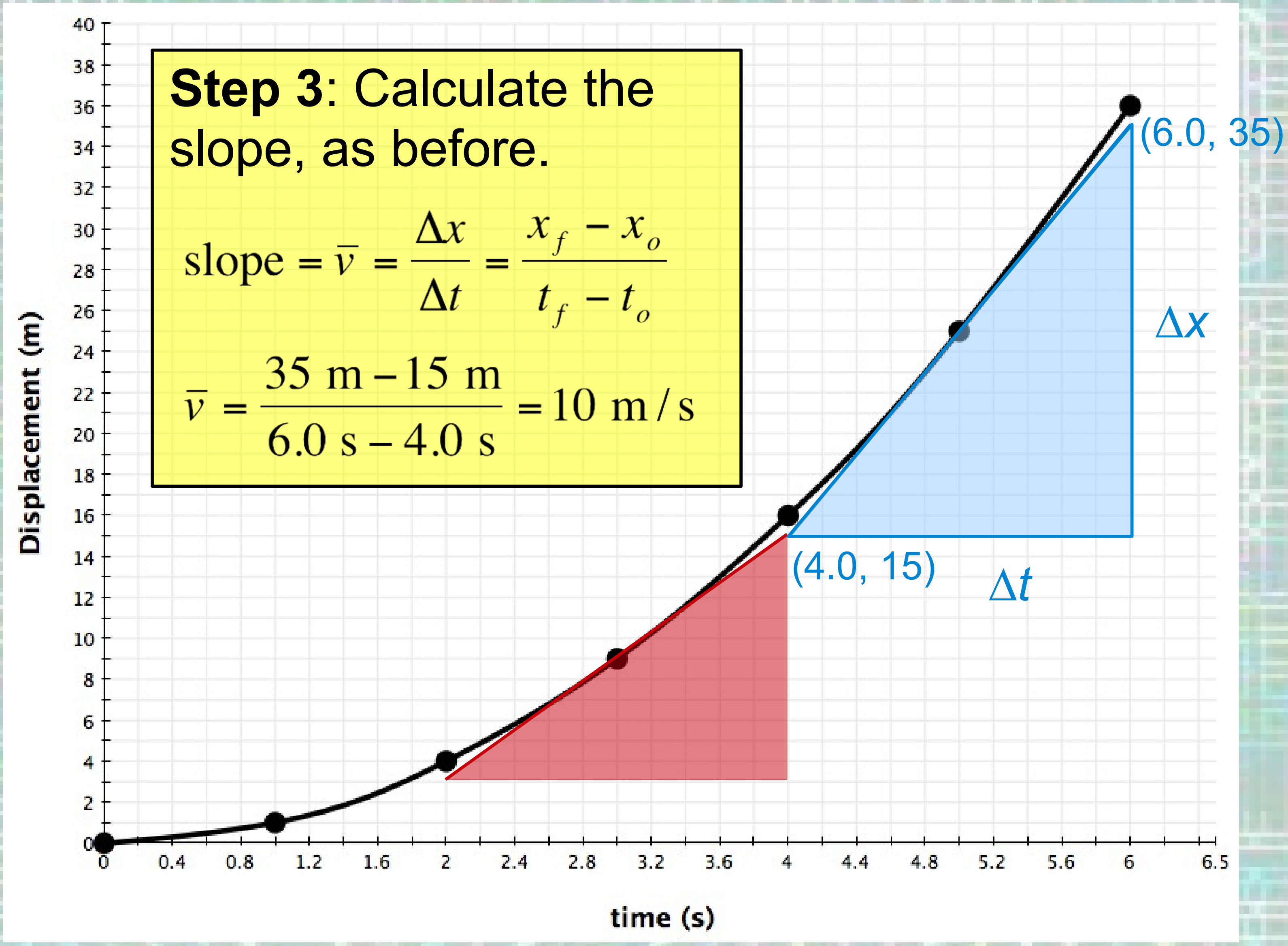
Finding the Velocity (Slope) at a Particular Point



Finding the Velocity (Slope) at a Particular Point

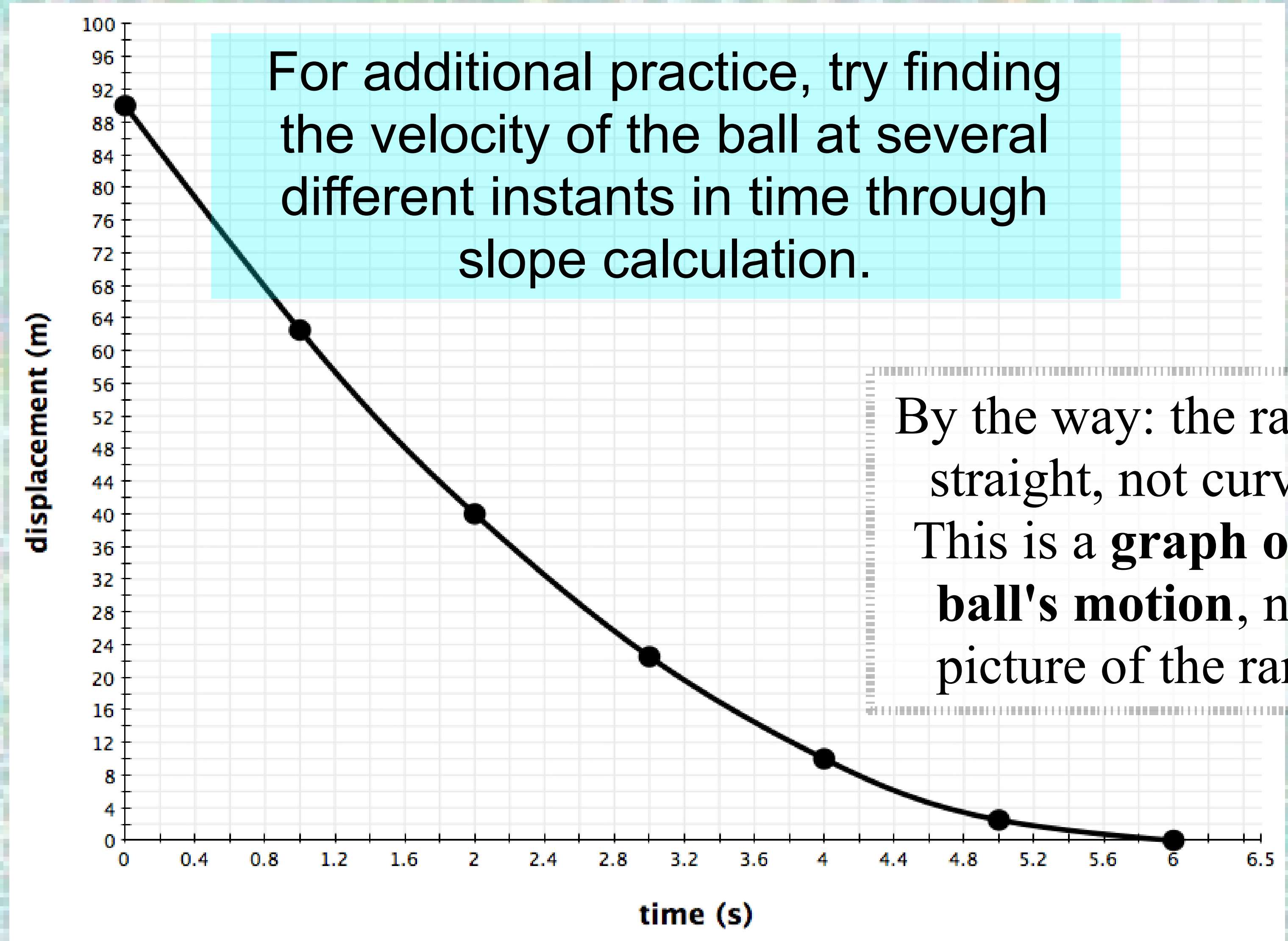


Finding the Velocity (Slope) at a Particular Point



Case Study #6

Imagine you're standing at the top of a ramp. Your friend rolls a ball up the ramp toward you. The following graph describes the motion of the ball.



For additional practice, try finding the velocity of the ball at several different instants in time through slope calculation.

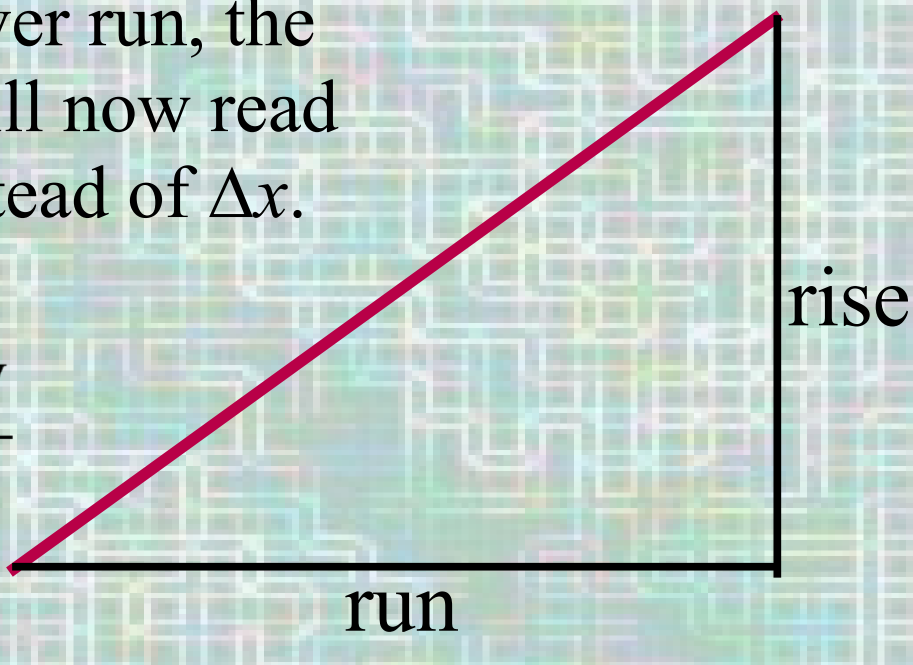
By the way: the ramp is straight, not curved. This is a **graph of the ball's motion**, not a picture of the ramp!

Velocity vs. Time Graphs

We will now remove displacement from the vertical axis of our graphs and replace it with velocity.

Although slope is still rise over run, the "rise" part of the equation will now read "change in velocity", Δv, instead of Δx.

$$\text{slope} = \frac{\text{change in velocity}}{\text{change in time}}$$



Slope as Acceleration

Our slope equation now becomes:

$$\text{slope} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o}$$

Since slopes can be either positive or negative, accelerations can be either positive or negative.

Speeding Up or Slowing Down?

Warning: Many people *mistakenly* believe that a positive acceleration means the object is speeding up and a negative acceleration means the object is slowing down.

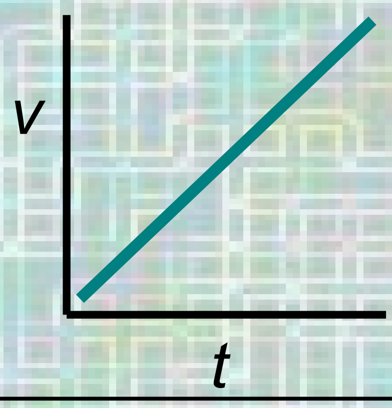
Here's the truth:

If the numerical values of an object's velocity and acceleration have the **same sign** (both positive or both negative), then the object is *speeding up*.

If the velocity and acceleration have **opposite signs** (one of positive and the other is negative), then the object is *slowing down*.

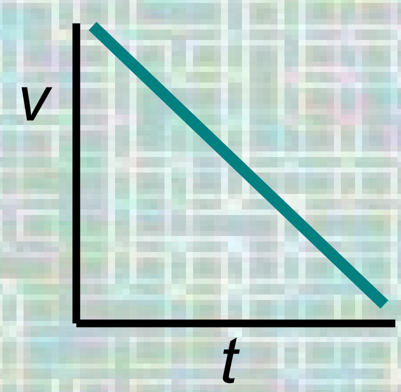
Here's How to Read that Information from Velocity vs. Time Graphs

Object speeding up



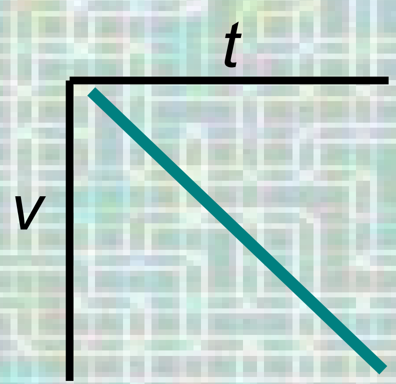
Velocity is +
(first quadrant)
Acceleration is +
(positive slope)

Object slowing down



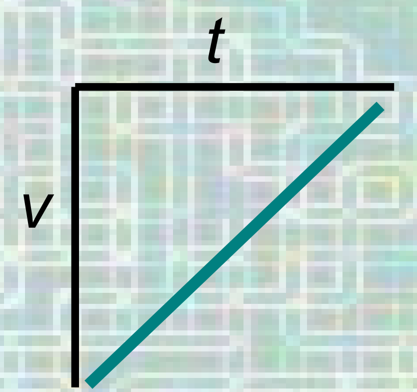
Velocity is +
(first quadrant)
Acceleration is –
(negative slope)

Object speeding up



Velocity is –
(fourth quadrant)
Acceleration is –
(negative slope)

Object slowing down



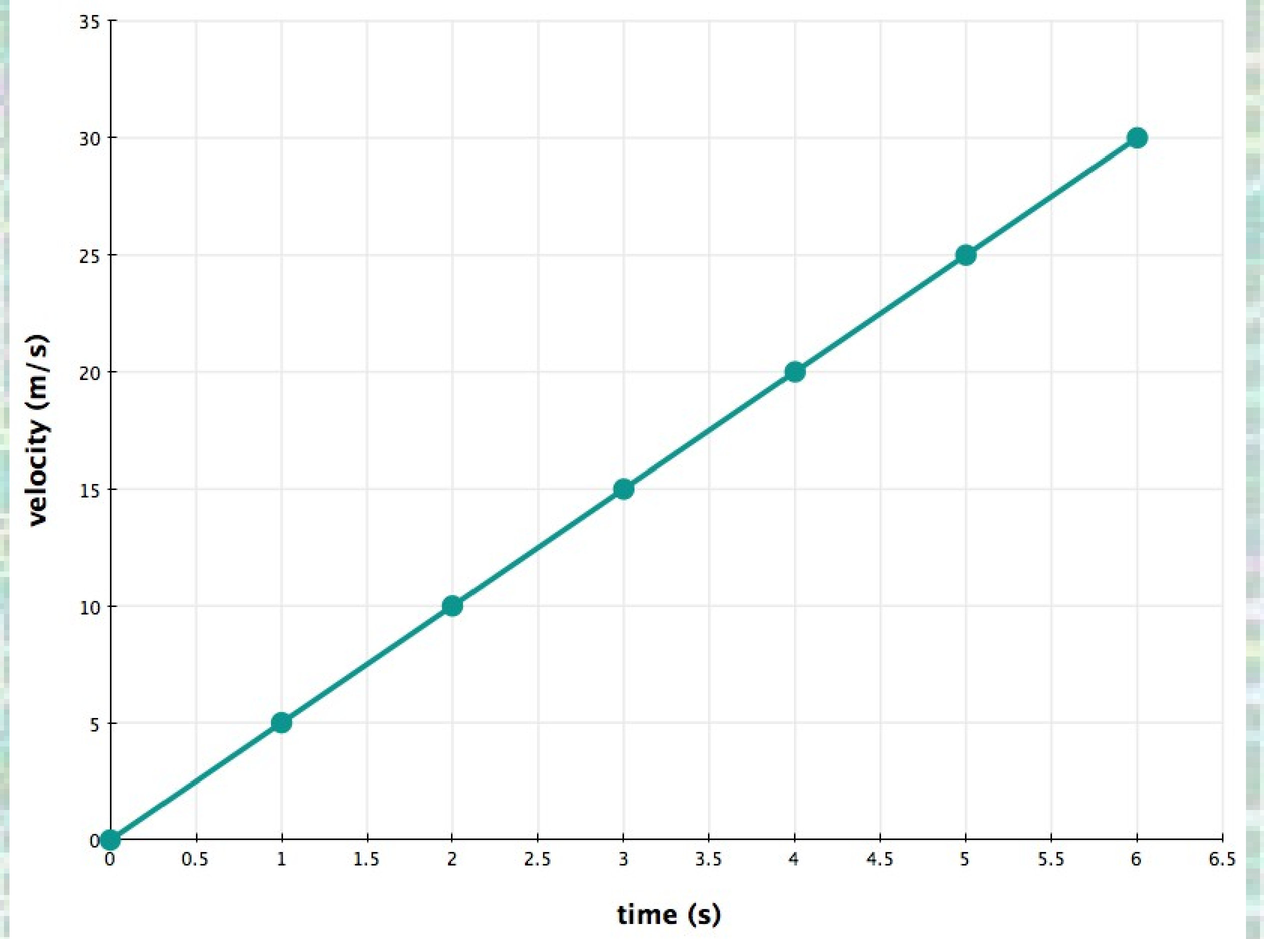
Velocity is –
(fourth quadrant)
Acceleration is +
(positive slope)

Case Study #7

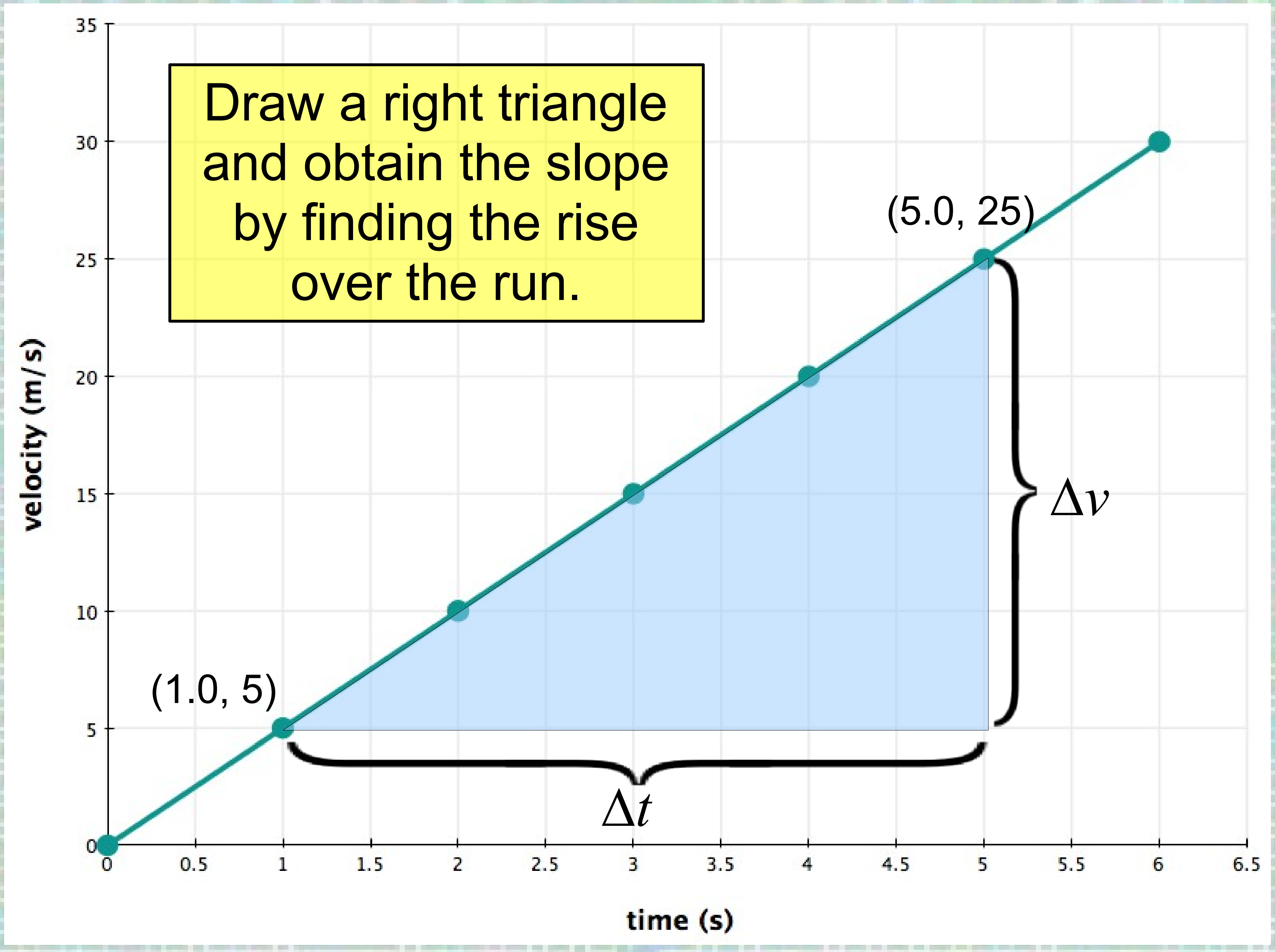
A bus has finished loading passengers, the door closes, and it accelerates away from the curb.

Since the graph line is straight, the bus has a constant acceleration.

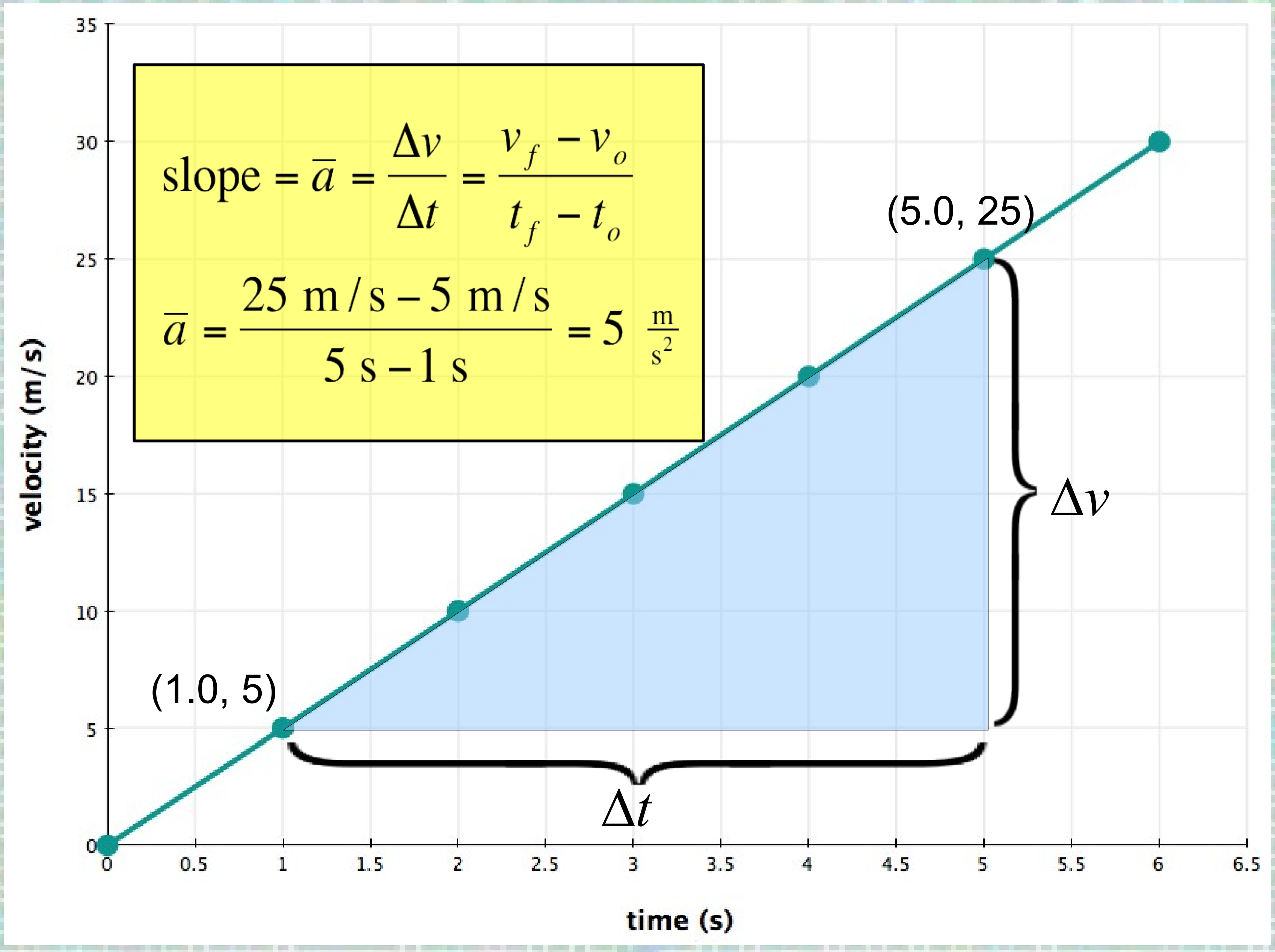
The value of the bus's acceleration can be calculated from the slope of the graph.



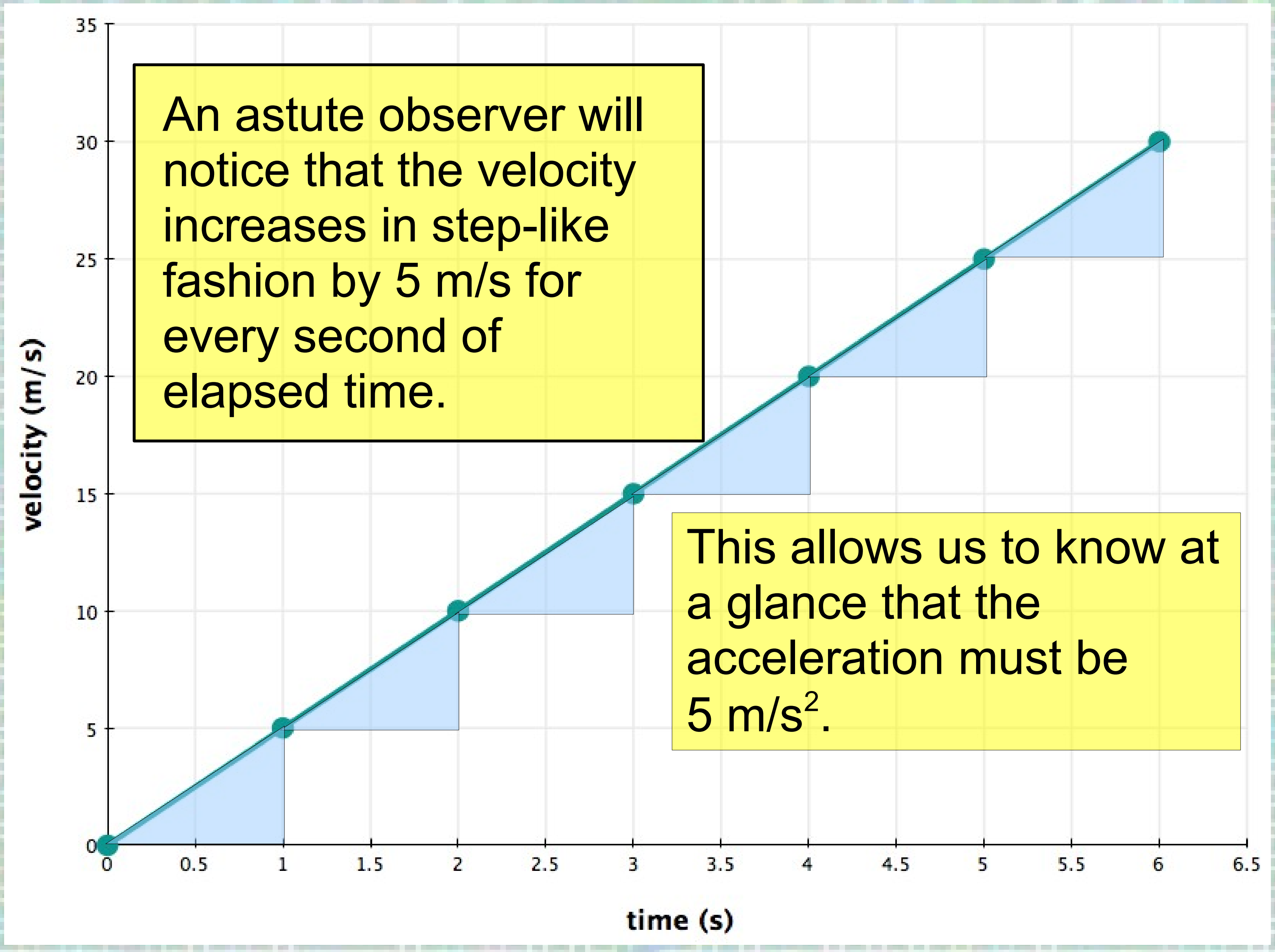
Obtaining the Acceleration from the Slope



Obtaining the Acceleration from the Slope



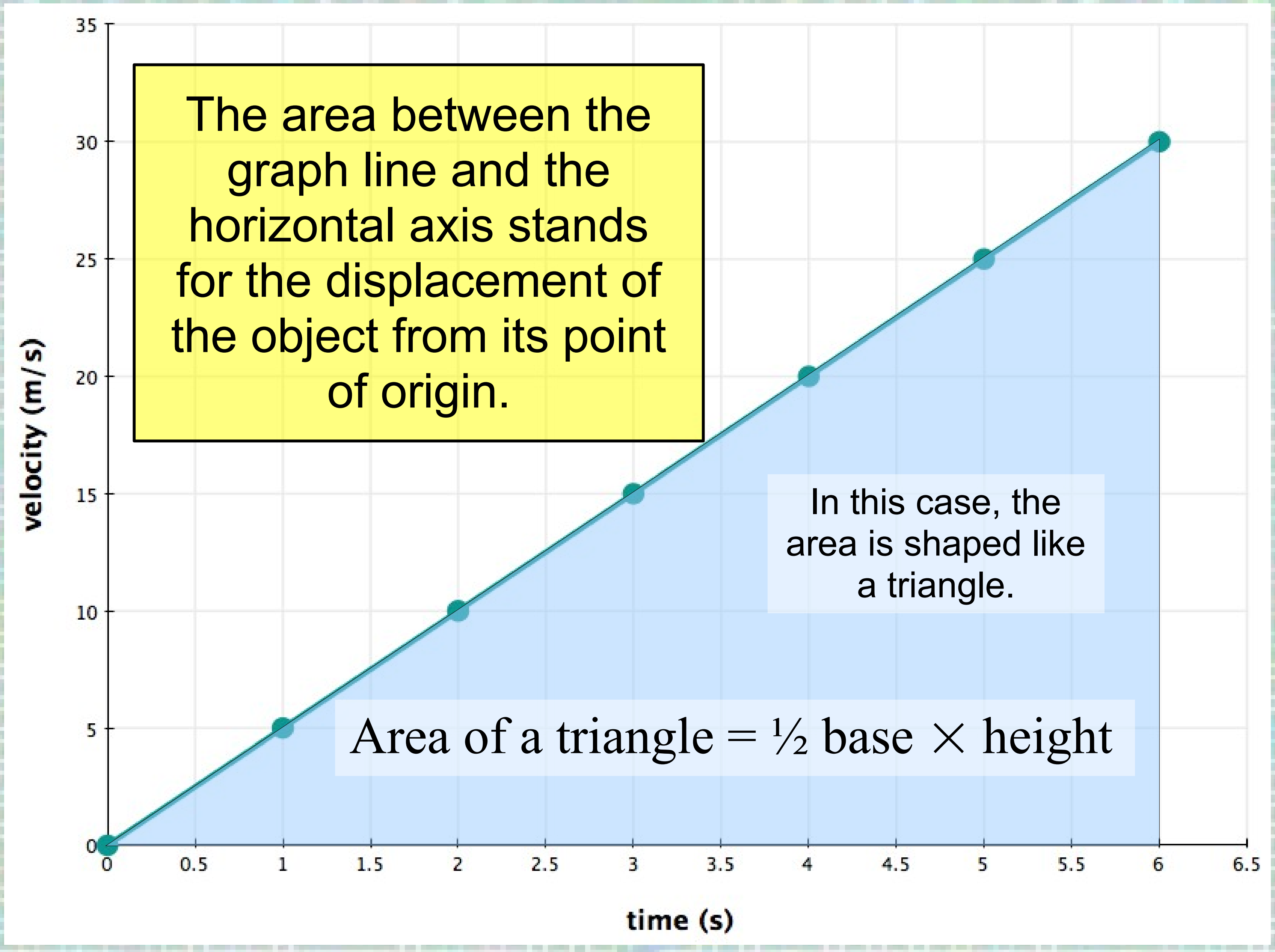
Obtaining the Acceleration at a Glance



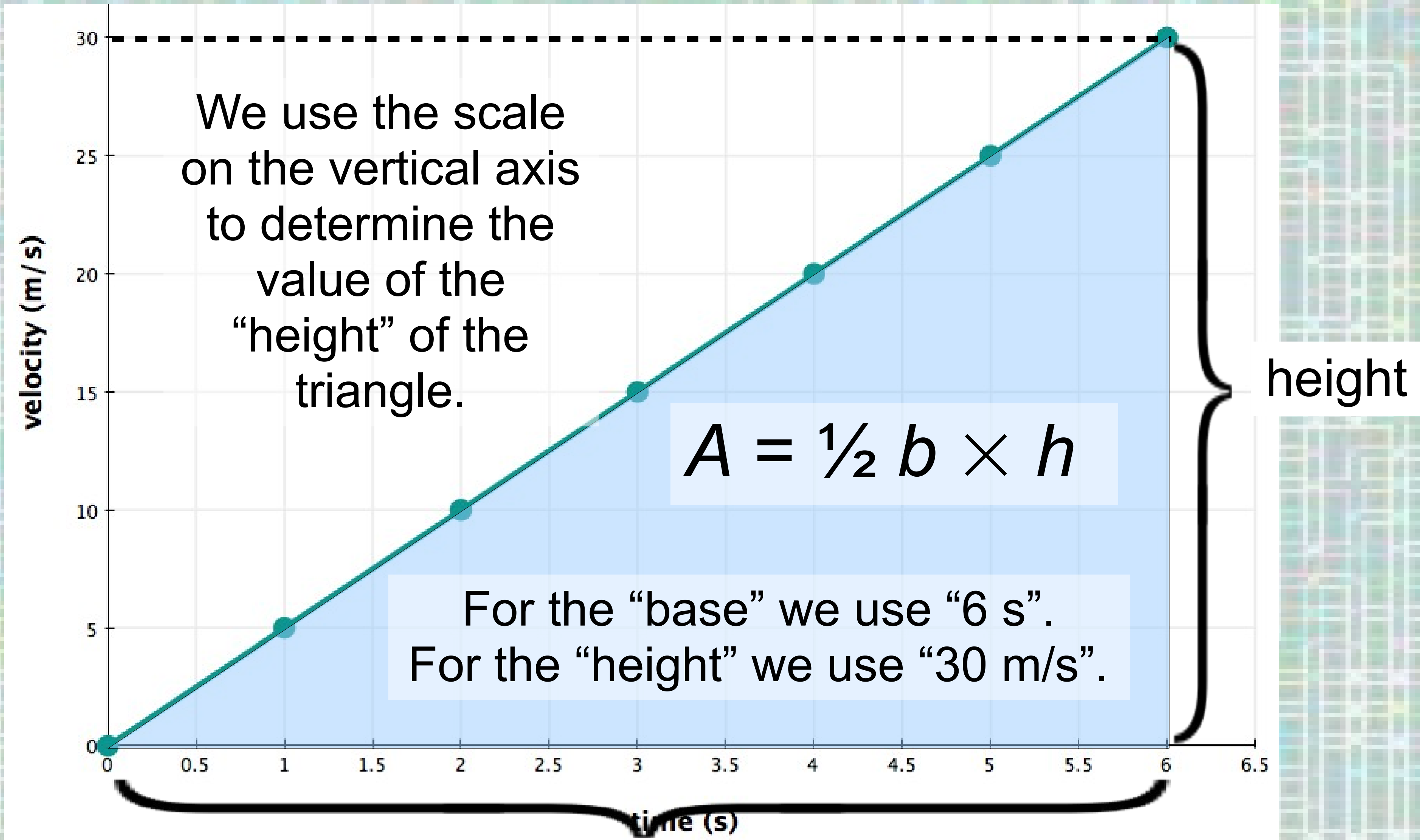
An astute observer will notice that the velocity increases in step-like fashion by 5 m/s for every second of elapsed time.

This allows us to know at a glance that the acceleration must be 5 m/s^2.

Displacement from Area

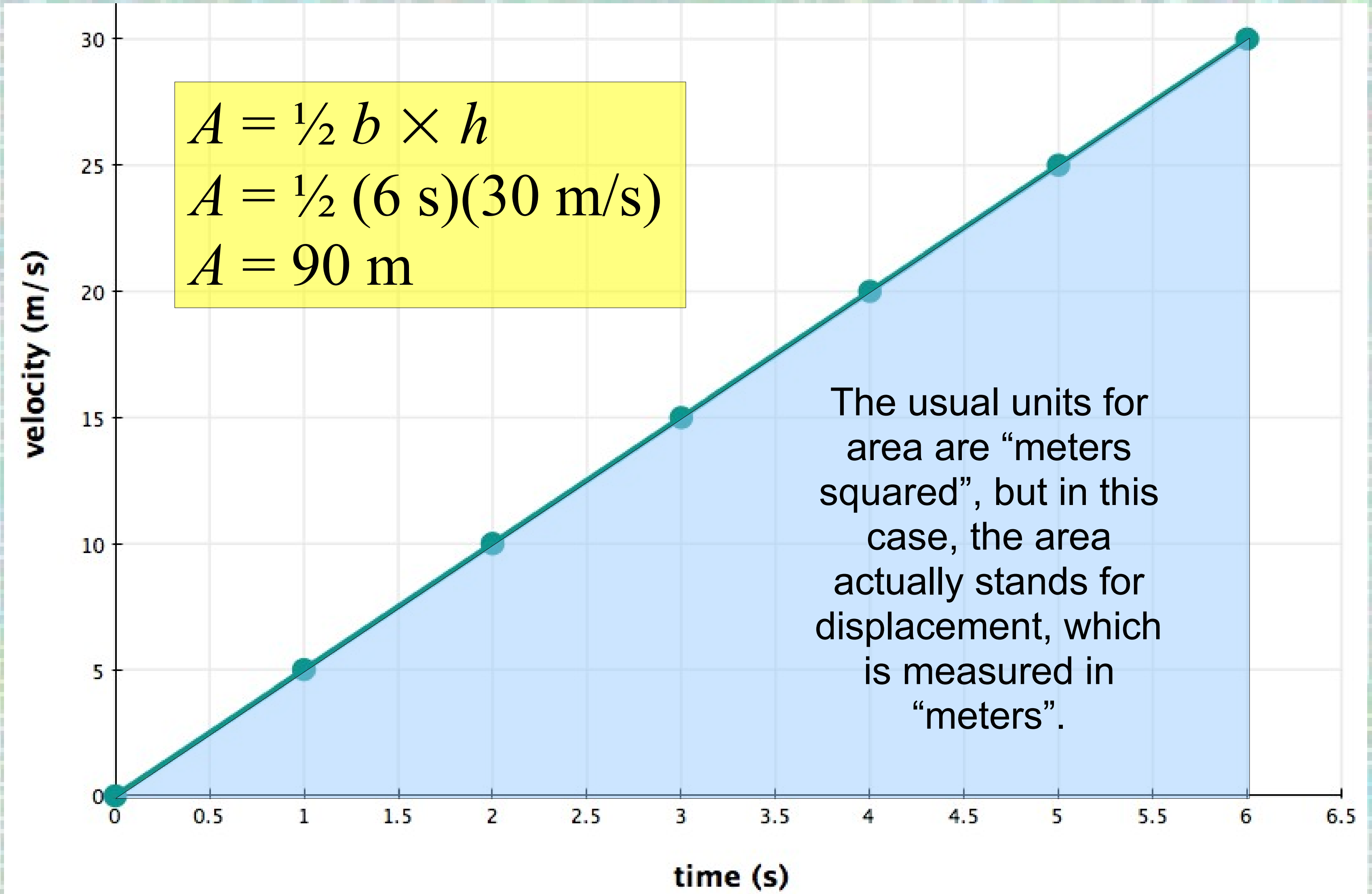


Displacement from Area



The scale on the horizontal axis is used to determine the value of the base.

Displacement from Area



This tells us that the bus has traveled 90 meters during the first 6 seconds since it left the curb.

Relationship Between Displacement and Time

Here is the equation we use to calculate the displacement of an object accelerating uniformly from rest:

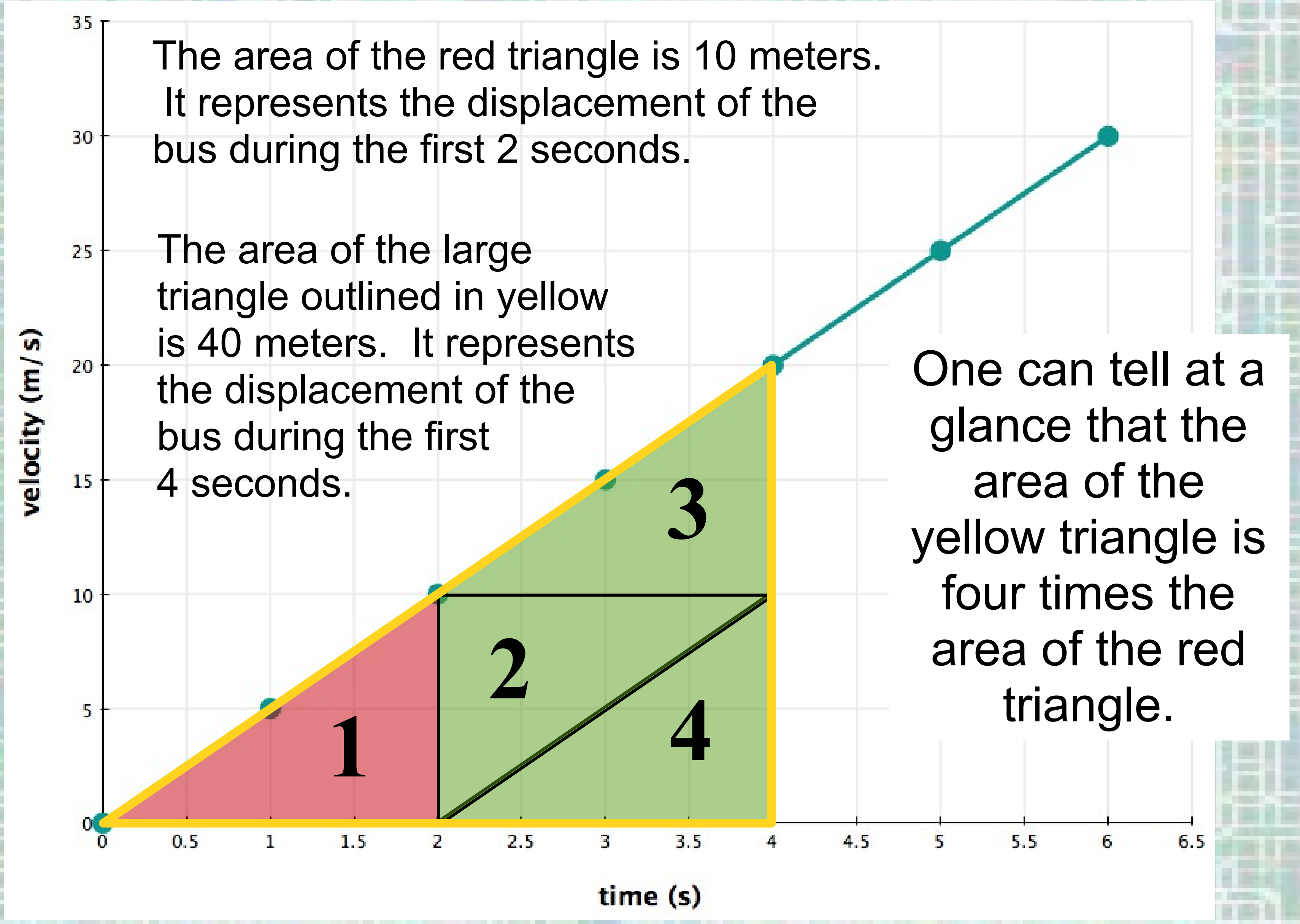
$$\boxed{\Delta x = \tfrac{1}{2}\, a\, t^2}$$

Expressed as a proportionality, it says that displacement is directly proportional to the square of the time.

$$\boxed{\Delta x \propto t^2}$$

This means that, in twice the amount of time, the object will be displaced 2^2 or four times as much.

Displacement ∝ Time Squared



Case Study #8

A policeman waits by the side of the road with a radar unit. At the instant a speeding car passes by, the police car begins to accelerate from rest in an effort to catch up to the speeder and pull him over.

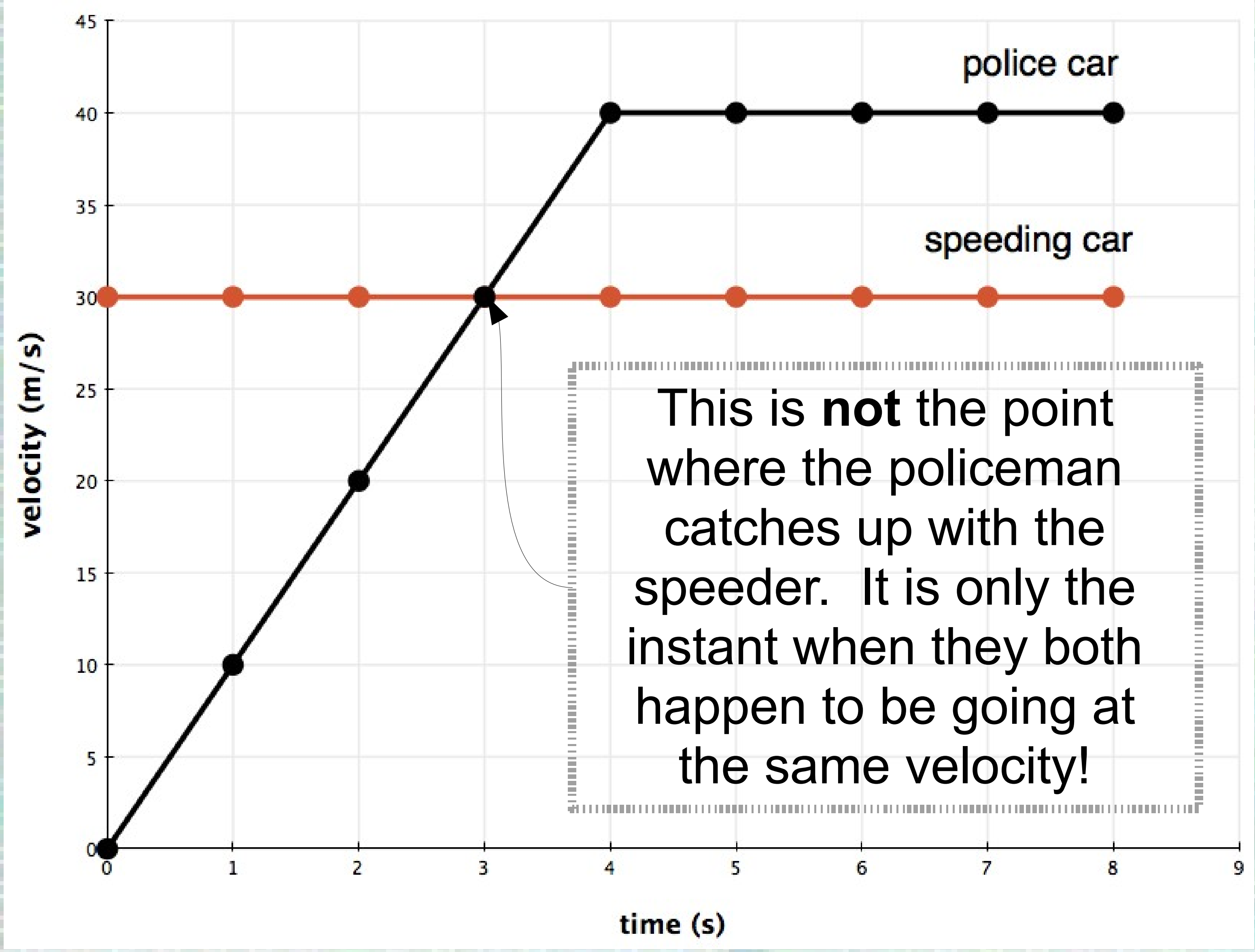
Here's the data:

time (s)	speeder's velocity (m/s)	cops's velocity (m/s)
0	30	0
1	30	10
2	30	20
3	30	30
4	30	40
5	30	40
6	30	40
7	30	40

Question:

At what time does the policeman catch up with the speeder?

The Cop and the Speeder

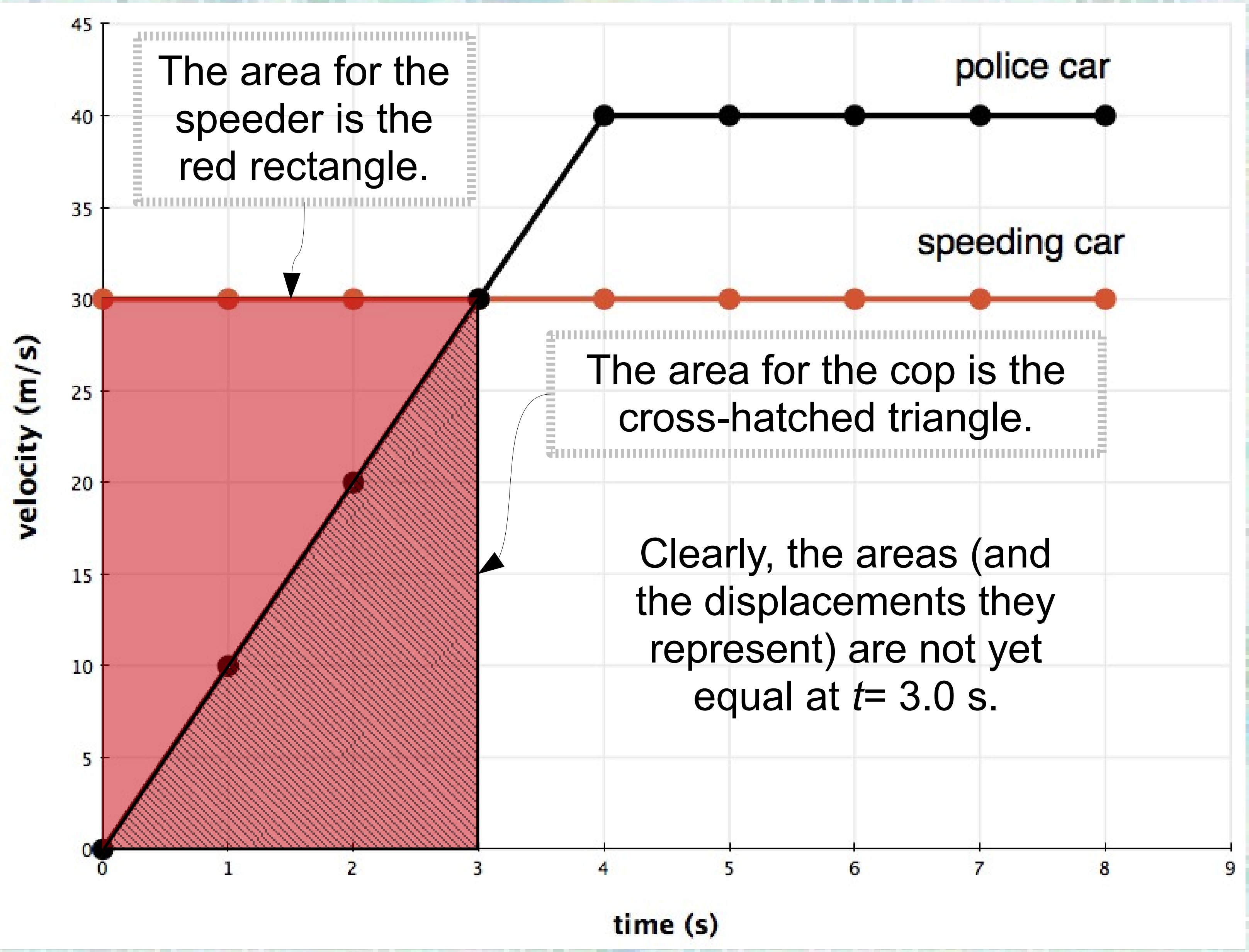


The Cop and the Speeder

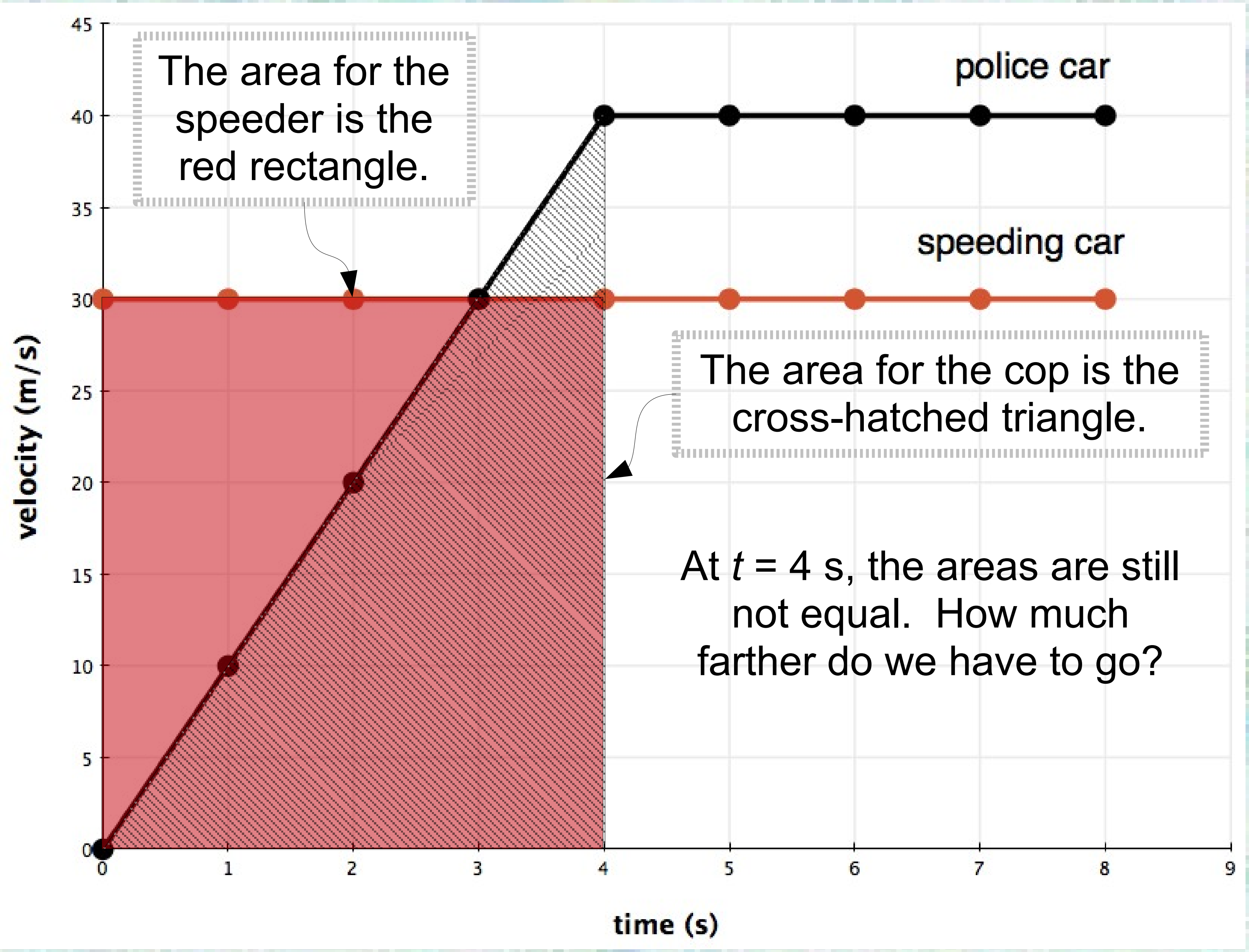
The true point where the cop catches up with the speeder is the point where they have both traveled the same distance forward from the point of origin (where the speeder first passed the cop when parked on the side of the road.)

You must find the moment in time when the **area under the graph line** for *cop* is the **same** as the **area under the graph line** for the *speeder*!

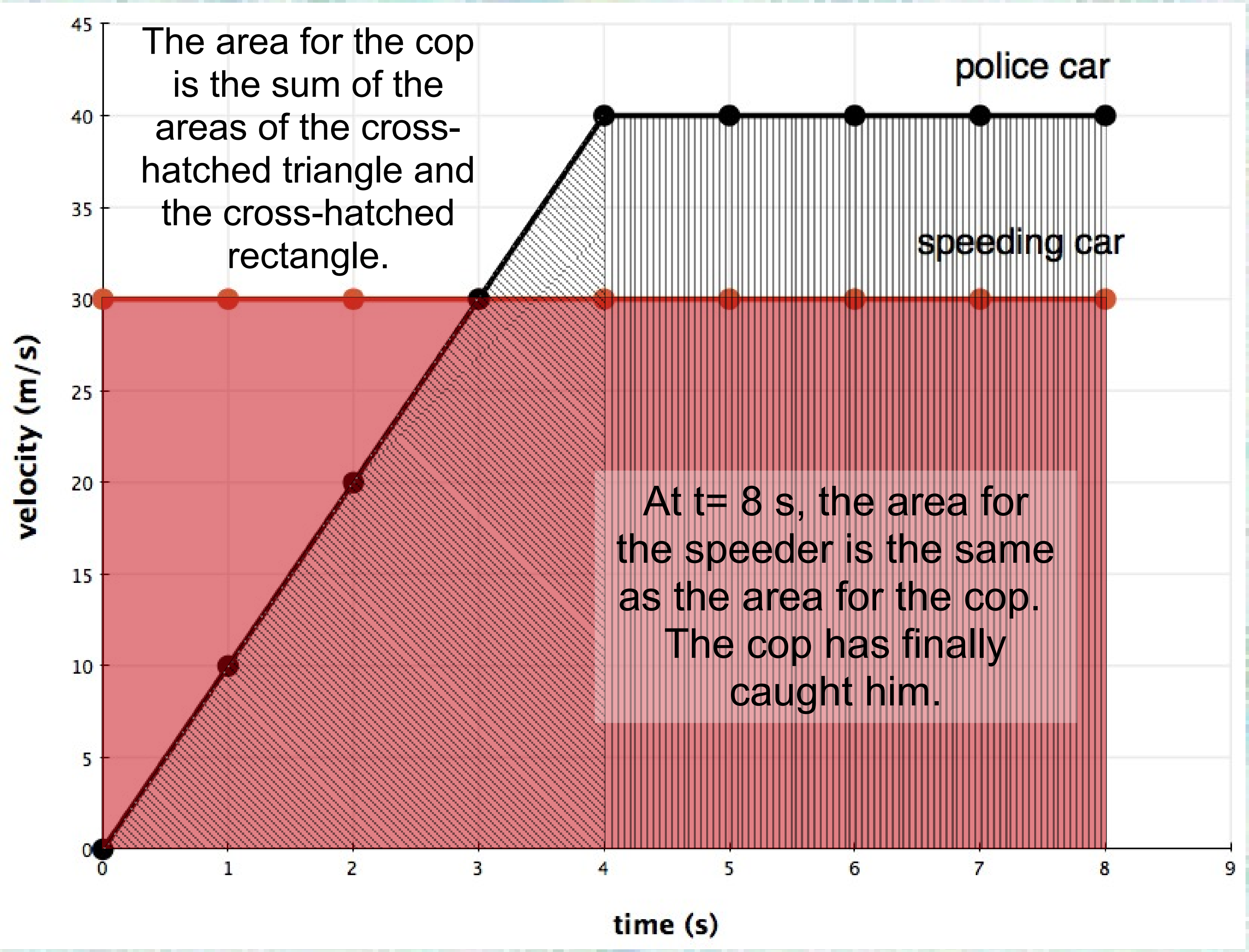
The Cop and the Speeder



The Cop and the Speeder



The Cop and the Speeder



The Cop and the Speeder

If it isn't obvious from the graph that the areas are the same, then you should do the calculation.

For the speeder:

$A = b \times h$

$A = (8.0\text{ s})(30\text{ m/s})$

$A = 240$ meters

Total displacement = 240 meters.

For the cop:

$A = \frac{1}{2}\, b \times h$ [for the triangle]

$A = \frac{1}{2}\,(4.0\text{ s})(40\text{ m/s})$

$A = 80$ meters

$A = b \times h$ [for the rectangle]

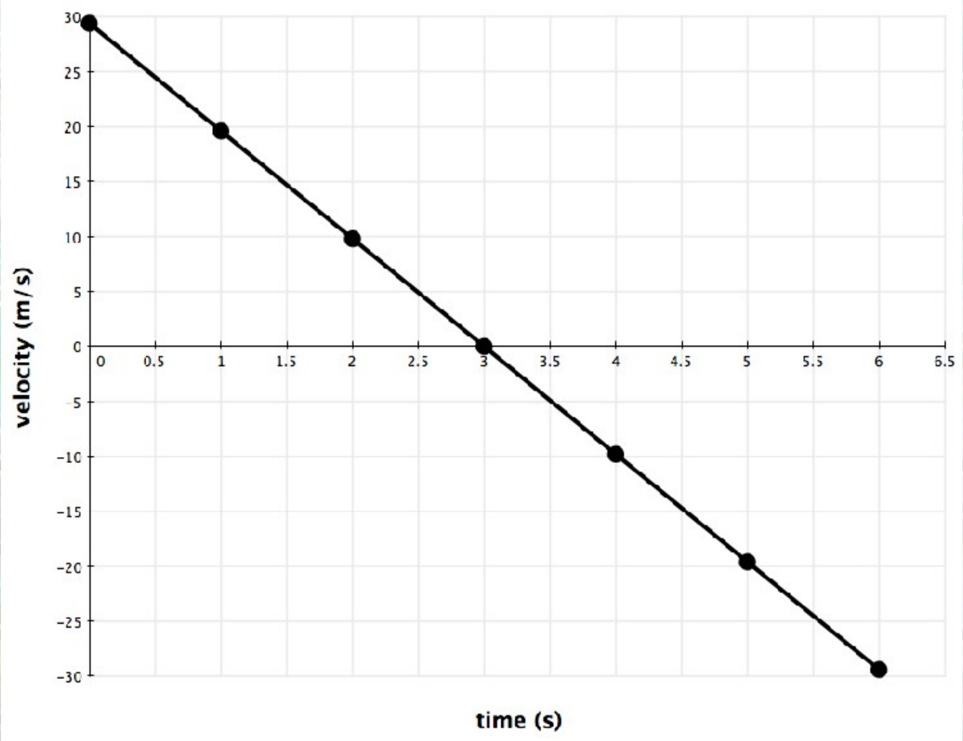
$A = (4.0\text{ s})(40\text{ m/s})$

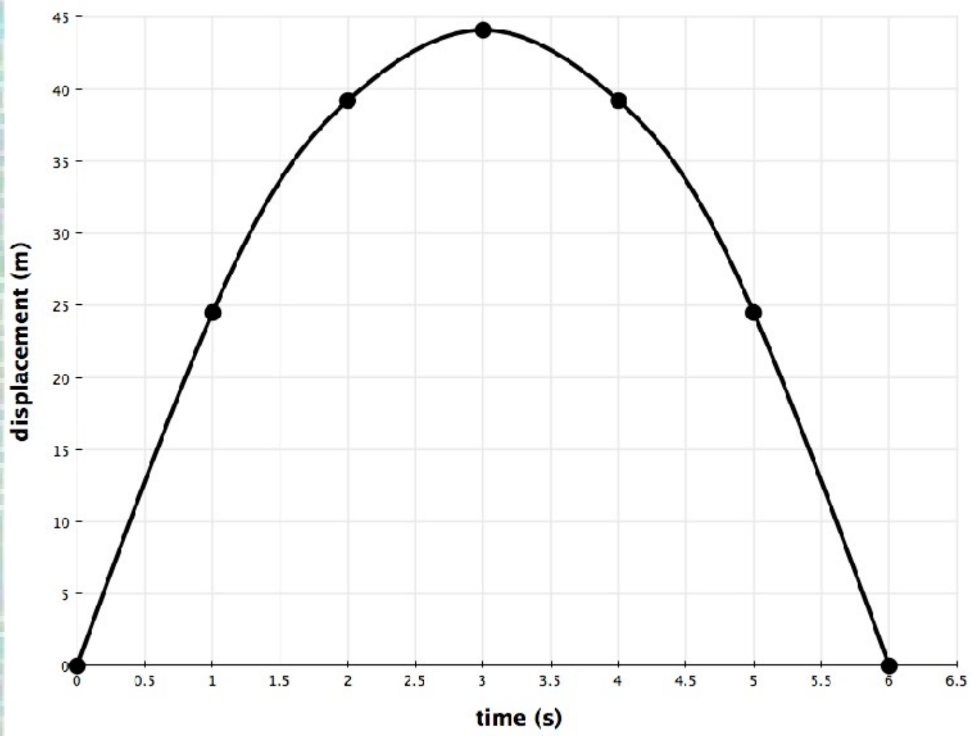
$A = 160$ meters

Total displacement = 80 m +160 m = 240 m.

Case Study #9

You throw a ball straight up in the air; it comes back down and you catch it. Here are the graphs that describe the motion of the ball.

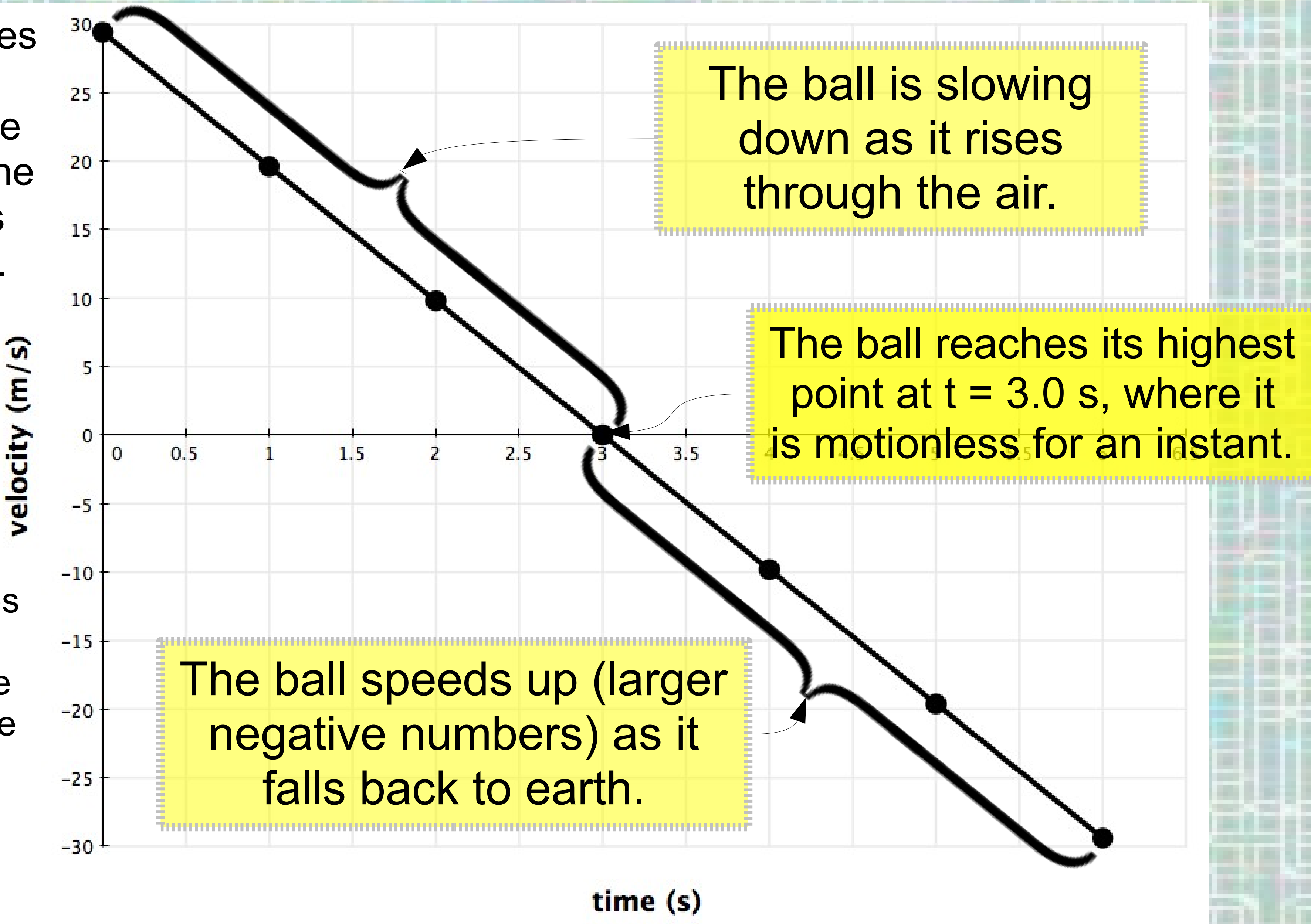




Both graphs describe the same motion!

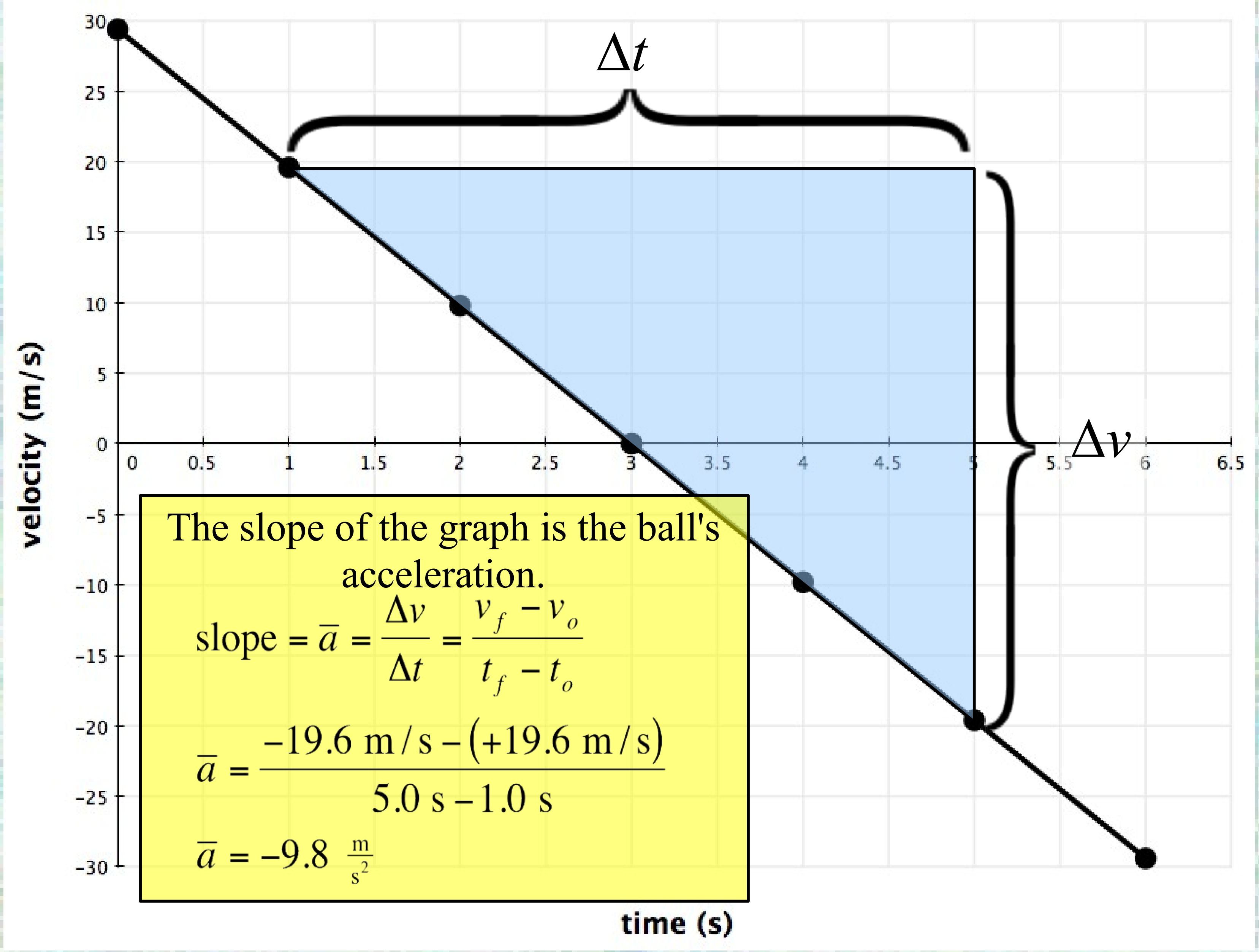
Characteristics of the *v* vs. *t* graph

Velocities are positive when the ball is rising.

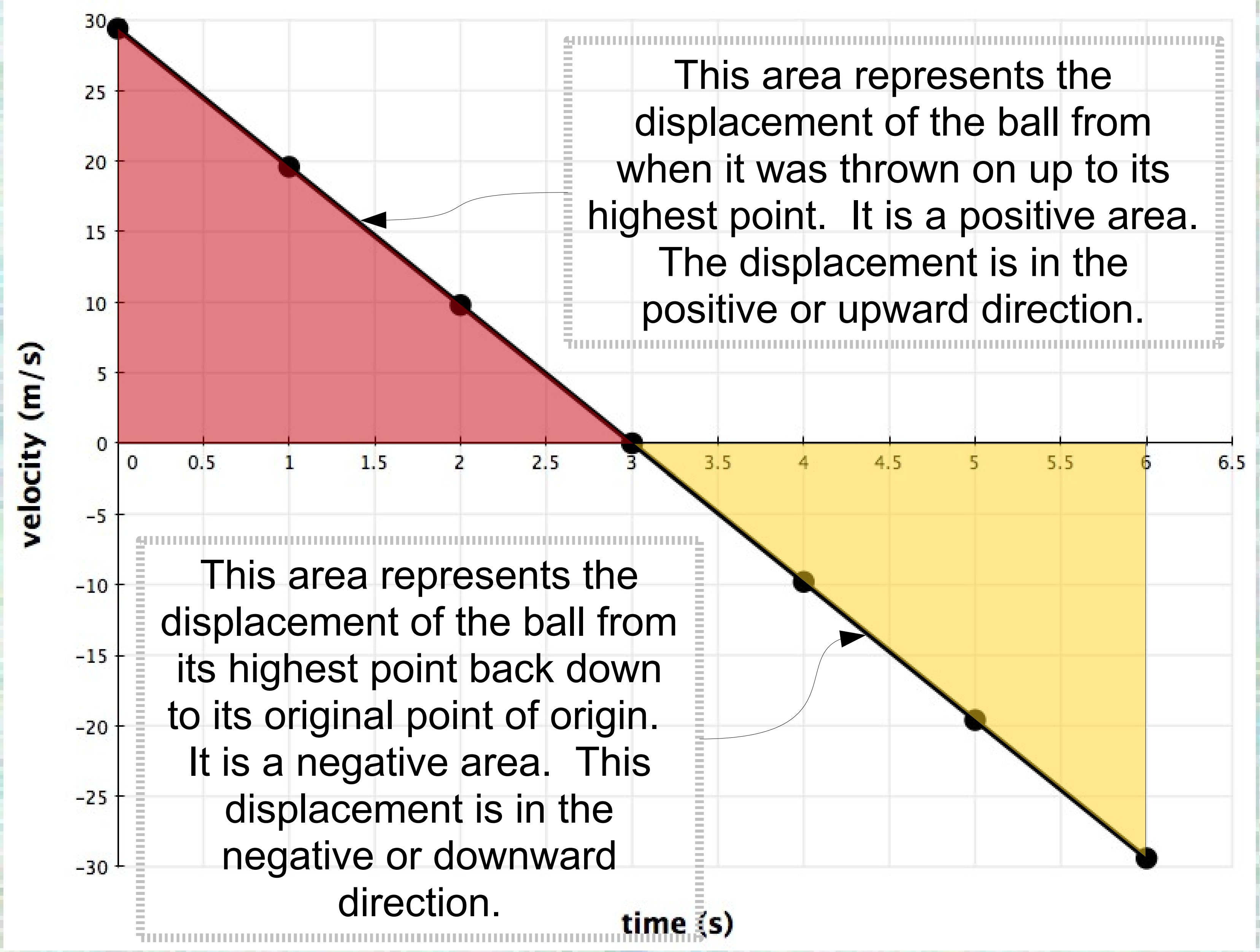


Velocities are negative when the ball is falling.

Characteristics of the *v* vs. *t* graph



Characteristics of the *v* vs. *t* graph



Characteristics of the *v* vs. *t* graph

Here's the upward displacement which occurs during the first 3 seconds.

$A = ½\, b \times h$

$A = ½\, (3.0\text{ s})(29.4\text{ m/s})$

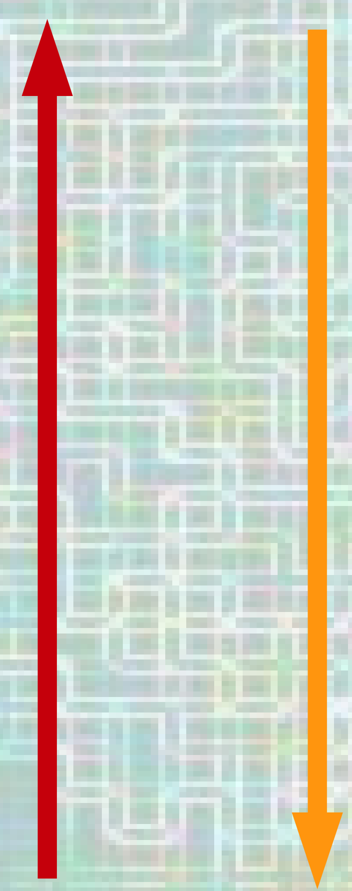
$A = +44.1$ meters

Here's the downward displacement which occurs during the last 3 seconds.

$A = ½\, b \times h$

$A = ½\, (3.0\text{ s})(–29.4\text{ m/s})$

$A = –44.1$ meters



The total displacement is zero. The ball ends up back where it started.

Characteristics of the *x* vs. *t* graph